Do Twelfths Terminate or Repeat?

Rebecca Ambrose and Erica Burnison

When we find the decimal equivalent of a fraction with 12 in the denominator, will it terminate or repeat? This question came from a seventh grader in author Erica Burnison's class as the student was pondering a poster generated by one of her classmates. Not only did we find the question intriguing, but it also affirmed our belief in the power of tailoring instruction to students' interests and needs.

As math educators, trying to figure out how to implement the content and Standards for Mathematical Practice (SMP) found in the Common Core State Standards for Mathematics in a student-centered fashion, we were thrilled by this moment. This question from a student also demonstrated that students can engage in the SMP while working in collaborative groups.

In this article, we will describe the activities that gave rise to this student-generated question. We will also provide our collaborative reflection of

While seventh-grade students group fractions, they learn more about decimals, in particular, and division, in general.

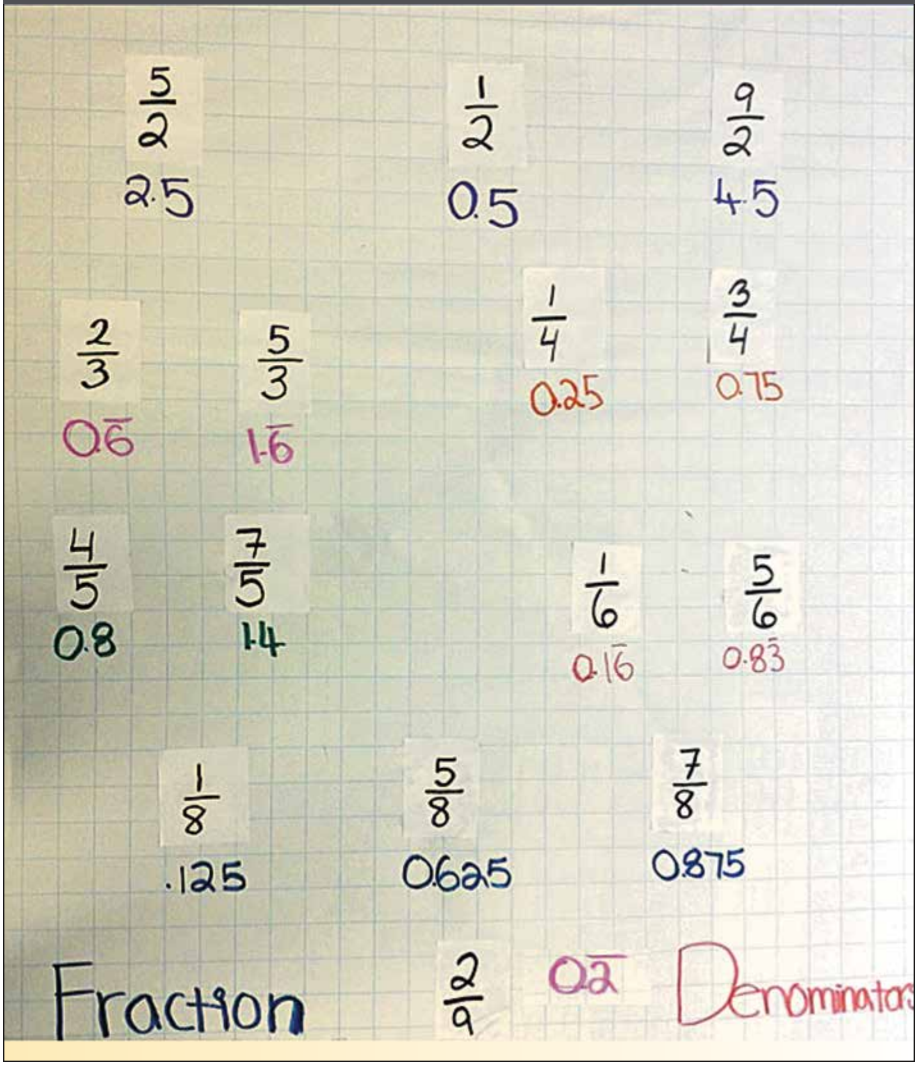
the activities because we find that collaboration deepens our understanding of pedagogy as well as mathematics. We hope that our account will inspire others to work together.

We work in a university-school partnership, Innovations in STEM Teaching, Achievement, and Research (I-STAR). Burnison is also involved with the BetterLesson Master Teacher Project, where she documented her teaching in 2013–14. These two projects gave her access to videorecordings of her students' conversations. Viewing videotapes of group discussions after class enabled her to eavesdrop on her students and use their ideas to tailor subsequent lessons to address their needs as well as meet the content standards.

INITIAL LESSON SET UP

The focus of the lessons we describe explores a grade 7 number standard in the Common Core: "Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats" (CCSSI 2010, 7.NS 2d; p. 49). Because Burnison's seventh-grade students typically exhibited a negative attitude about their math abilities and tended to be fearful of anything related to rational numbers, she anticipated knowledge gaps. She wanted to challenge her students' misconceptions, so she began by asking them to convert the fraction 5/8 to a decimal. When they asked, "Does

Fig. 1 Students were asked to produce equivalent fraction and decimal pairs from the set below, then sort them in any way they pleased.



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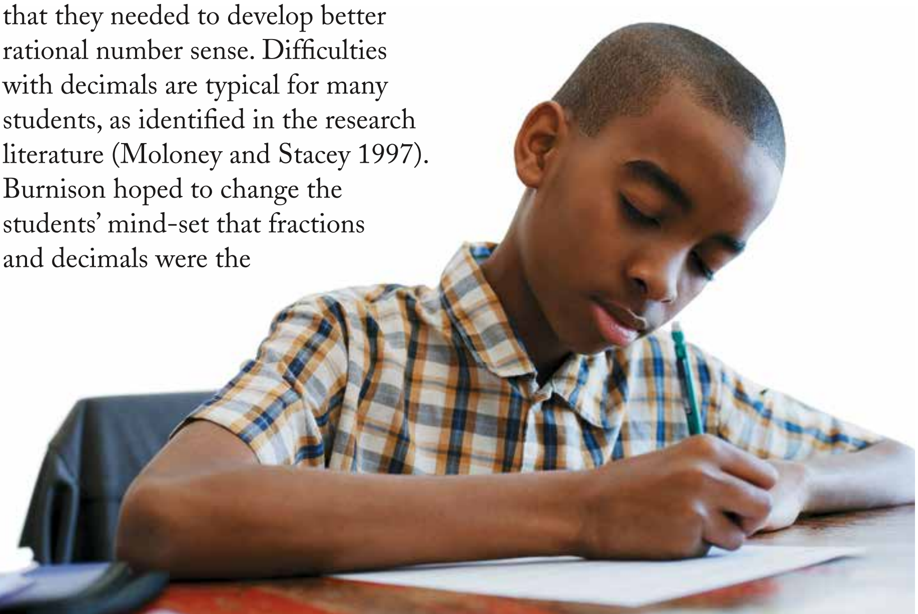
the 5 or the 8 go in the house?" and did not know whether 0.625 or 1.6 was a reasonable answer, she knew that they needed to develop better rational number sense. Difficulties with decimals are typical for many students, as identified in the research literature (Moloney and Stacey 1997). Burnison hoped to change the students' mind-set that fractions and decimals were the enemy by providing them with an opportunity to explore rational numbers in a nonthreatening way.



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Armed with the information about students' misconceptions, she created a set of fractions for small cooperative groups to use in a sorting task. She strategically chose fractions in the hopes that students would observe patterns that might spark conjectures and questions (see **fig. 1**). She wanted to hear her students talk about "What if" and "Why" rather than "How to" as they developed a better understanding of the connection between fractions and decimals. She also hoped the task would help them confront some of their misconceptions.

To complete the task, students converted each fraction into a decimal and then used a calculator to double-check their work. Once they had the equivalent fraction and decimal pairs, students were asked to sort the fractions into sets in any way that made sense to them. Burnison did not care how they chose to sort them; she wanted them to look beyond the procedures that seemed to defeat them and instead experience the excitement of discovery that she hoped would emerge from noticing patterns.

Students represented their sorting discoveries on posters (see **fig. 2**). Some groups sorted the fractions according to same denominators; others separated repeating and terminating decimals; and others distinguished between fractions that were more or less than one whole. The variety of responses provided fodder for good discussions in future lessons, so the effort to tailor instruction to student needs could continue. At this point in the unit, students had experienced using division to convert fractions to decimals. Their engagement with the SMP had only just begun.

ONE GROUP'S INTERACTION

Burnison examined the posters and decided that she would provide prompts to engage students in examining one another's work and

[The teacher] chose specific fractions because she wanted to hear "What if" and "Why" rather than "How to."

thinking about what classmates had completed. The prompts helped students use the organizational patterns created by their classmates to explore the Common Core's content standard.

The prompt that went with the poster in **figure 2** was this:

> Is there anything you notice about the fractions that tells you in which place value the decimal ends or how the decimal ends?

These students had grouped all the fractions with the same denominator. In viewing the arrangements, Sam's, Elena's, and Juana's attention was drawn to the fractions with 8 in the denominator. They claimed that "when you divide by 8, it always ends in the thousandths place and it always ends in a 5." At this point, the trio had already noticed that fractions with 8 in the denominator convert to decimals that terminate with a 5 in the thousandths place. Sam continued to look for additional examples to support their claim, but, finding none, moved on to the next pattern.

In retrospect, we realized that an opportunity was missed to suggest to the students that they generate their own data by providing more examples (i.e., 2/8 or 3/8) and testing their claim; however, we were happy that at least Sam realized that more data would strengthen his argument.

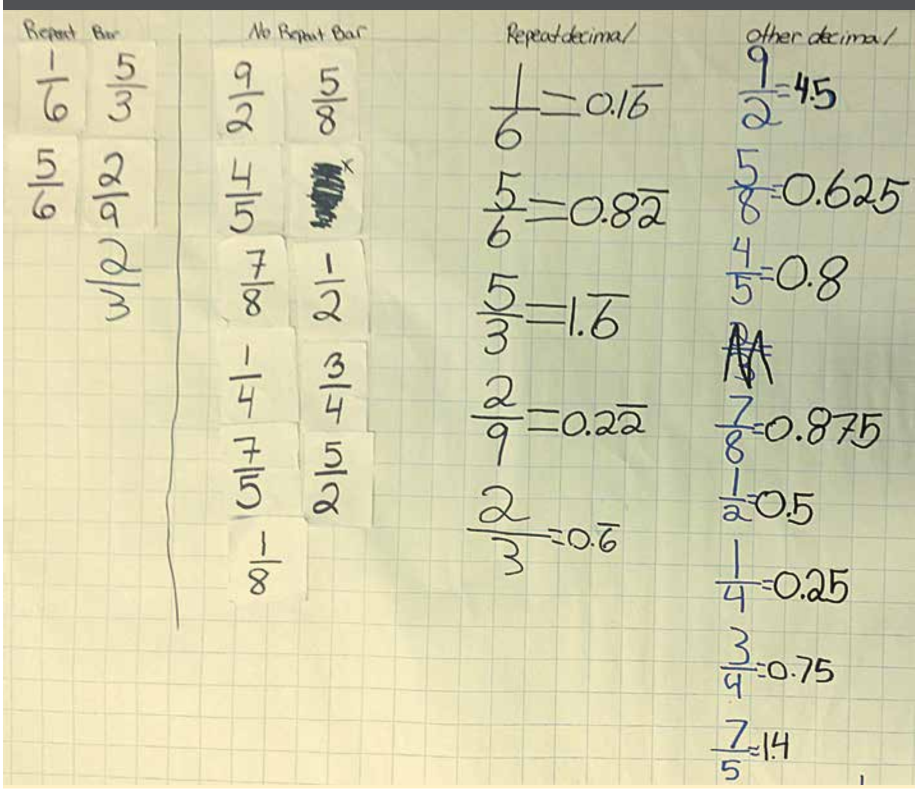
Elena then asked the group, "What about dividing by 2, 4, or 6?" They scanned the poster again and noticed that dividing by 2 also results in a decimal that "ends in a 5." Elena then pointed to 5/2 = 2.5, 9/2 = 4.5, and 1/2 = 0.5 to illustrate her point. Sam repeated the conjecture that "when you divide by 2, when the denominator is 2, it ends in a 5." These students had recognized another pattern in the relationship between the denominator and the terminating decimal equivalent, but had not asked why it occurred. More observations ensued.

Elena: Well, dividing by 2 and 4, it will end in a 5, too.
Sam: [Continued focusing on fractions with 2 in the denominator] These are all tenths. [Then he pointed to 3/4 = 0.75.] But this one ends in the hundredths, and the eighths ends in the thousandths. They all have 5.
Juana: Not all of them.
Sam: [Revising his statement] Yeah, the higher the number it is, the higher the denominator; well, it has to be 2, 4, 8.
Elena: Wait, are you saying all of them? All of them end in 5?
Sam: Only 2, 4, and 8 all end in 5, but 8s always in the. . .
Juana: Thousandths.

Sam and Juana then stated the rest of their claim together as they pointed to examples on the poster, "Twos always in the tenths, and fourths are in the hundredths."

Elena was particularly persistent in her critique by asking which fractions Sam was talking about, causing Sam to engage in SMP 6, "Attend to precision," as he specified which fractions ended in a 5.

Fig. 2 It was hoped that students would find patterns while they were sorting fractions.



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ANOTHER GROUP'S INTERACTION

Another group had created a poster that separated the fractions into categories of repeating and nonrepeating decimals. Burnison decided to use this poster, illustrated in **figure 3**, to help students explore the patterns in repeating decimals and the division that produced them.

This group noticed that all the denominators for the repeating decimals were multiples of 3 (i.e., 3, 6, and 9). When Adam listed the denominators out loud, he did not stop at the examples provided on the poster but extended his pattern to denominators of 12. Isabella immediately questioned his assertion and pointed out that 12 is a multiple of both 3 and 4. She wondered if, when converting fractions to decimals, a denominator of 12 would act more like 3 and repeat or more like 4 and terminate ending in 5. Burnison was delighted that Adam extended the pattern and that Isabella critiqued his reasoning.

Both groups of students had noticed and expressed regularity (SMP 8) and had identified and used the structure of the number system (SMP 7). The first group noticed two regularities: The fractions whose equivalent decimal ended in 5 had denominators of 2, 4, and 8 and, within that group of fractions, the number of digits after the decimal point increased by 1 as the denominator doubled. The second group noticed that all the repeating decimals had fraction equivalents with a multiple of 3 in the denominator. Moreover, they were extending the pattern to fractions with a denominator of 12 and conjecturing about how its decimal equivalents would behave. By articulating their observations to one another, they were working toward SMP 3, "Construct a viable argument and critique the reasoning of others." Although their arguments could use some fine-tuning, we were excited to see them engaging independently in the Common Core's Standards for Mathematical Practice. They understood Burnison's expectation that they make sense of things for themselves and share their ideas with classmates, and their interactions indicated that they were wholeheartedly participating in the activity. Their discussions provided Burnison with a window into their thinking and another opportunity to tailor her instruction to their needs.

Fig. 3 This listing demonstrated why denominators with powers of 2 convert to decimals that end in a 5.

$$\frac{1}{2} = \frac{1}{2} \bullet \frac{5}{5} = \frac{5}{10} = 0.5$$

$$\frac{1}{4} = \frac{1}{2^2} = \frac{1}{2^2} \bullet \frac{5^2}{5^2} = \frac{5^2}{10^2} = \frac{25}{100} = 0.25$$

$$\frac{1}{8} = \frac{1}{2^3} = \frac{1}{2^3} \bullet \frac{5^3}{5^3} = \frac{5^3}{10^3} = \frac{125}{1000} = 0.125$$

STUDENTS' QUESTIONS

The next day, the class was told about the question that Adam and Isabella had brought up, "What happens when fractions with 12 in the denominator are turned into decimals?" and were invited to figure it out for themselves. Burnison designed an activity sheet to help gather data, listing all the twelfths from 1/12 through 12/12. The students were eager to get started trying to answer their classmates' question. After simplifying the fractions, most students used the division algorithm to convert the fractions to decimals. Burnison was pleased to see that students were using their knowledge of the size of the fraction to decide how to set up the division, a procedure that initially had thwarted them.

In one group, Marianna suggested that all the fractions that could be simplified would be converted to repeating decimals. However, Francisco and Becka pointed to the fractions 3/12, 6/12, and 8/12 as counterexamples to Marianna's claim. The group persevered and continued to look for patterns in the data. Like most of the groups, these students concluded that the fractions, once written in lowest terms, did hold the key to deciding what denominators of 12 would do. Those that simplified to a denominator of 3 would convert to repeating decimals, and those that simplified to denominators of 2 or 4 would terminate with a 5 in the tenths and hundredths place, respectively.

Another group made a slightly different observation, asserting that if a factor of 4 was removed when simplifying, the resulting decimal would repeat. Alternatively, if a factor of 3 was removed, the fraction would convert to a terminating decimal. Students were discovering relationships among fractions, decimals, and division and engaging in the Standards for Mathematical Practice.

Burnison was so focused on accomplishing this goal that she missed an opportunity to dig deeper into the math when Miguel shared his thinking. While completing the activity sheet, Miguel realized that he did not need to use long division to convert fractions to decimals, but instead could think about equivalent fractions with powers of 10 in the denominator. He did not articulate this pattern, but noted that 3/4 = 75/100 = 0.75. In this case, Miguel had found a useful structure in the numbers (SMP 7). He stepped back and shifted perspective to notice the equivalent fractions embedded in the process of converting fractions to decimals. Burnison did not immediately appreciate this

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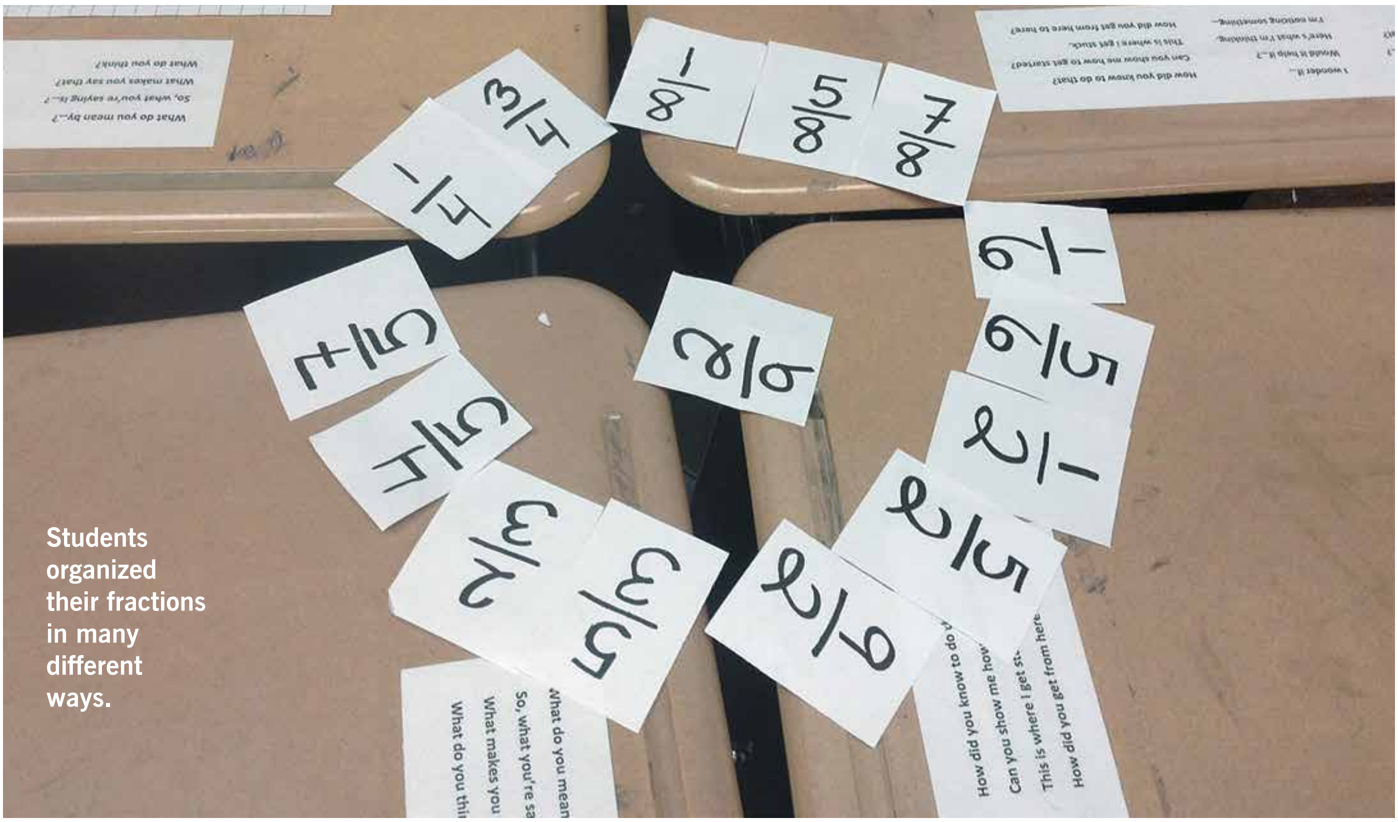
underlying big idea in the midst of instruction.

Sometimes it is difficult to recognize these opportunities during class and make adjustments in the moment. After we, the authors, collaborated, the important mathematical idea that Miguel was approaching became clear. We realized that the students were looking for patterns but were not asking what was causing them. In particular, they did not provide a justification for why fractions with a power of 2 in the denominator (2, 4, 8) would terminate in a 5 when converted to a decimal.

Although we discussed the math in **figure 3**, we are not sure if seventh graders could have generated this analysis using exponents. Even so, Burnison appreciated the discussion because she realized that thinking about equivalent fractions in relation to converting fractions to decimals would allow her to extend her students' thinking should the opportunity arise. She recognized that emphasizing decimals as a "special kind of fraction with a power of 10 in the denominator" would help her students understand why some fractions terminated and why others did not. In retrospect, she wished that she had engaged the whole class in thinking about Miguel's approach so she could see how far they could go with it (see Kreith 2014 for a more extensive discussion of **fig. 3**).

FROM DIVISION PHOBIA TO UNDERSTANDING

Burnison's students had initially been division-phobic and were lacking number sense with respect to fractions and decimals. Some teachers might avoid exploring the relationship between fractions and decimals until after students had mastered long division and after students understood fractions better. Instead, Burnison had students explore terminating and repeating decimals; in so doing, it bolstered their abilities in division and their understanding of fractions simultaneously. Although Burnison addressed several practice standards, she continually adjusted instruction to take advantage of her students' contributions. She used the posters they produced as a resource and presented a student-generated question for all to investigate. She made it clear that



Students organized their fractions in many different ways.

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she valued their work and found their ideas worthy of consideration. In response, students engaged in several SMP, actively looked for patterns, searched for structure, articulated their thinking, and attended to precision.

She was able to tailor instruction to her students' needs and input, meet the content standard, and facilitate the math practices because she had faith in her students' curiosity and in their intelligence. In addition, she explicitly valued their thinking. She had nurtured a classroom culture in which students felt comfortable sharing their thinking and questioning each other.

Students' conversations fueled their thinking, and together they came up with interesting ideas to pursue. This series of lessons felt like happy accidents that followed from students' ideas and questions. But really they were a result of lessons purposely designed to elicit student thinking which, in turn, provided further avenues for exploring math.

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Any thoughts on this article? Send an email to **mtms@nctm.org**.—*Ed.*

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