Fostering Perseverance

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Sustaining engagement with a mathematics task is not a novel suggestion for effective mathematics teaching. *Principles and Standards for School Mathematics* (2000) specified that "students need to know that a challenging problem will take some time and that perseverance is an important aspect of the problem-solving process and of doing mathematics" (NCTM 2000, p. 186). Yet there is often an expectation in mathematics class that solutions should come quickly, and teachers work against a norm that has equated fast responses with proficiency in mathematics. The Common Core's Standards for Mathematical Practice (SMP) (CCSSI 2010) have brought renewed attention to the importance of perseverance in mathematical problem solving. In this article, we offer a set of classroom practices that foster students' perseverance in problem solving.

For a seventh-grade classroom, a group of teachers (see the acknowledgment at the end) designed instructional moves to foster perseverance in a lesson. Students spent a full class period working on a single challenging problem that stumped everyone at first. In fact, at the outset, students seemed lost, and it might have been tempting for the teacher to step in and show students what to do. But this lesson, by design, provided time and structure for the students to arrive at solutions themselves. At several junctures along the way, students huddled excitedly in groups to share their progress, even though they were far from finding complete solutions. Through a series of carefully formulated questions and participation structures, students demonstrated perseverance and enthusiasm. This did not happen by accident.

With the goal of designing instruction to foster student perseverance, a group of teachers met multiple times to first choose a suitable mathematical task and then develop a lesson. In so doing, the teachers were anticipating students' strategies and responses as well as developing language for responses to support students' abilities to maintain "productive struggle" (Hiebert and Grouws 2007; NCTM 2014; Warshauer 2015) on a challenging problem. Our team of teachers and professional developers then observed one team member teach the lesson. They met together afterward, in a modified form of "lesson study" (Stigler and Hiebert 1999), to analyze how it went. Although qualities like perseverance are not built in a day, the lesson was designed to specify teacher

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moves that could be woven into daily instruction. Here we share five categories of those teaching moves from that lesson that constitute teachers' efforts to foster student perseverance in problem solving:

Perseverance can be developed only by using tasks that warrant perseverance. Task selection is key.

1. Selecting mathematical tasks that require and support perseverance

For this lesson, we considered features of a worthwhile task, as defined by the National Council of Supervisors of Mathematics (NCSM 2013). According to NCSM, a great task—

- revolves around an interesting problem—offering several methods of solution;
- is directed at essential mathematical content as specified in the standards;
- requires examination and perseverance—challenging students;
- begs for discussion—offering rich discourse on the mathematics involved;
- builds student understanding—following a clear set of learning expectations; and
- warrants a summary look back—providing reflection and extension opportunities.

Teachers selected the Party Time problem (see **fig. 1**) as a good candidate for inviting seventh-grade students' perseverance in problem solving. They anticipated that this problem could not be solved quickly because it involved no computation and had little resemblance to other problems recently solved, so that students could not rely on a pattern from their prior work to solve it. This problem definitely was interesting for the students and, as you will see below, students had multiple ways of approaching the problem. This open-ended problem was also chosen for its potential for discussion and reflection. Perseverance can be developed only by using tasks that warrant perseverance. Task selection here was key.

2. Talking about strategies for problem solving

Theresa Drauch taught the lesson, designed in collaboration with her colleagues, to her seventh-grade students. Throughout the lesson, she prompted students to talk about strategies that they might use to solve the problem, especially when they were stuck. When the problem was first distributed, a full four minutes passed before students attempted to write anything on their papers. After giving students some time to work on the problem, Drauch opened a discussion about the strategies they employed:

T: What strategy were you using?
S1: I made a list. [See **fig. 2a**.]
S2: I constructed a table.
S3: I wrote down costumes with equals signs next to them. [See **fig. 2b**.]
S4: I circled the challenging ones. [See **fig. 2c**.]
T: Work on it again, use others' strategies, and see if that will help you.

CCSSI (2010) states that students need to "monitor and evaluate their progress and change course if necessary" (p. 6), and teachers designed supports to name and encourage this kind of experimentation. In an exit ticket, a student described how she changed strategies as she worked (see **fig. 3**).

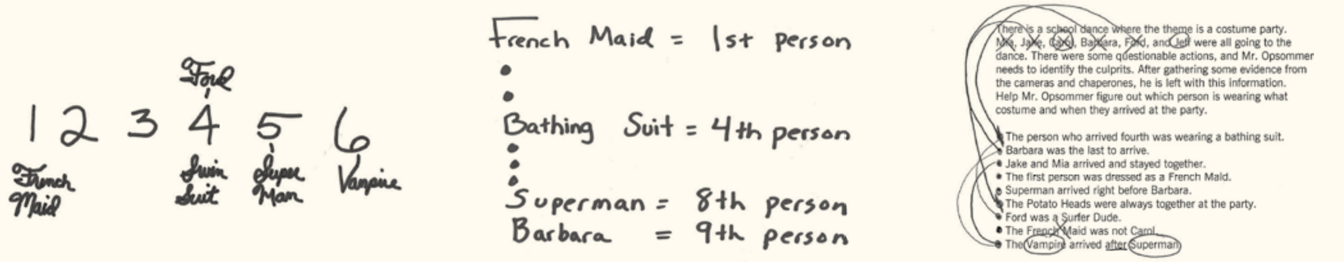
Fig. 1 Teachers chose the Party Time mathematics problem after considering characteristics of a worthwhile task.

There was a school dance where the theme was a costume party. Mia, Jake, Carol, Barbara, Ford, and Jeff went to the dance. There were some questionable actions, and Mr. Opsommer needed to identify the culprits. After gathering some evidence from the cameras and chaperones, he was left with this information. Help Mr. Opsommer figure out which person was wearing what costume and when he or she arrived at the party.

- The person who arrived fourth was wearing a bathing suit.
- Barbara was the last to arrive.
- Jake and Mia arrived and stayed together.
- The first person was dressed as a French Maid.
- Superman arrived right before Barbara.
- The Potato Heads were always together at the party.
- Ford was a Surfer Dude.
- The French Maid was not Carol.
- The Vampire arrived after Superman.

Source: Adapted from http://www.insidemathematics.org/assets/problems-of-the-month/party%20time.pdf

Fig. 2 Examples of students' initial work included several strategies.



(a) Making a list **(b) Writing equal signs next to costumes** **(c) Circling the challenging costumes**

3. Demarcating phases in the problem-solving process

This lesson unfolded in phases that Drauch delineated with questions, directives, and alternating participation structures. These demarcations slow down the problem-solving process and encourage perseverance by staving off the rush to obtain a solution, thus making patient, deliberative work the norm in solving a challenging problem.

Students were prompted to talk about what they had figured out before they had complete solutions, and the teacher used language to signal the notion that problem solving takes time and that this work is valuable as one presses on toward a complete solution. Below are three different questions or directives that the teacher offered at different stages of the lesson:

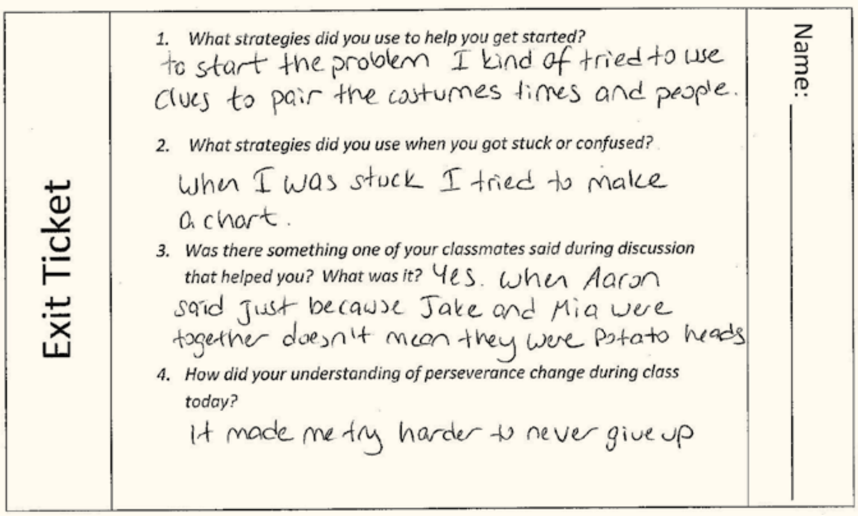
- Share with someone near you something you discovered while you were working that really helped you and that you would use again.
- Describe one thing to your partner, but explain your thinking.
- Pair with someone who has a different strategy than yours and confer.

Note that these directives explicitly prompt students to leverage the thinking of peers in solving this challenging problem. Other teacher moves affirm the time it takes to solve problems and the fact that dead ends are to be expected:

- What did you do when you got stuck?
- Where are you [in the process right now]?

Teachers also segmented the flow of the lesson with alternating participation structures. As part of their lesson design, the teachers planned how much time would be devoted to each component of the lesson to ensure that all would be included. The lesson design began with individual work so that students could gather their thoughts on their own, signaling that each student was able to generate useful ideas for solving the problem. This notion is key to fostering perseverance. But teachers also allocated time following that segment of individual work time for students to confer in pairs, so that once they had a solution path on their own they could gain other ideas from a partner and help one another get "unstuck." The next participation structure was

Fig. 3 The student who completed this exit ticket described how she changed her strategies as she progressed through the problem.



Exit Ticket

1. *What strategies did you use to help you get started?*
to start the problem I kind of tried to use clues to pair the costumes times and people.
2. *What strategies did you use when you got stuck or confused?*
When I was stuck I tried to make a chart.
3. *Was there something one of your classmates said during discussion that helped you? What was it?* Yes. When Aaron said just because Jake and Mia were together doesn't mean they were Potato heads
4. *How did your understanding of perseverance change during class today?*
It made me try harder to never give up

Name:

group work, to expand the exchange of ideas among students. Finally, students participated in whole-class discussion, armed with ideas that had been fleshed out in the earlier participation structures. Although the teachers allotted a certain number of minutes for each segment, Drauch also had the latitude to change the participation structure when students needed to switch gears—if she sensed that many had run into a blind alley in their problem-solving efforts, or, conversely, when they discovered a useful tidbit that would aid in solving the problem.

4. Naming feelings attendant to problem solving

The teacher in this lesson named feelings that were often attendant to problem solving, especially problem solving that demanded perseverance. She had students offer ways of managing some of those feelings.

T: How many of you got frustrated? [Many students raise their hands.] So, what did you do about that?
S1: I kept trying.
S2: I used SMP 4.
S3: I used SMP 1 and 6.
S4: I used SMP 8: looked for a pattern.
S5: I tried something different.
T: When we were solving a problem last week, a lot of students got frustrated along the way. What did people do about that?
S6: Keep trying.
S7: Try something different.

Here the teacher has invoked prior experiences, and in response, students referenced the Standards for Mathematical Practice that helped them move forward. Copies of these standards hang from the ceiling around the room on laminated pages that students access frequently during class. Earlier in the school

Fig. 4 Additional exit tickets were handed out at the end of the lesson.

Exit Ticket

Name:

1. What strategies did you use to help you get started?
I used something like a pattern and I read the paper over and over.
2. What strategies did you use when you got stuck or confused?
I looked back, started all over then I got my answer.
3. Was there something one of your classmates said during discussion that helped you? What was it?
One of my classmates said "Just because Jake and Mia were together does not mean they were the potato heads."
4. How did your understanding of perseverance change during class today?
My understanding of perseverance changed when someone helped with learning my mistakes.

(a)

Exit Ticket

Name:

1. What strategies did you use to help you get started?
The strategies were: putting in a line with the kids on one side and the costumes on the other and made a line to the ones I thought matched.
2. What strategies did you use when you got stuck or confused?
The strategies I used when I got stuck were to stop and think.
3. Was there something one of your classmates said during discussion that helped you? What was it?
Yes, there was something my classmate helped me during the discussion. It was when the connected the Surfer Dude and Bathing Suit.
4. How did your understanding of perseverance change during class today?
My understanding of perseverance changed during class today was how my friends wouldn't let me give up.

(b)

Exit Ticket

Name:

1. What strategies did you use to help you get started?
I did matching and connecting.
2. What strategies did you use when you got stuck or confused?
I did mental math and more matching.
3. Was there something one of your classmates said during discussion that helped you? What was it?
No
4. How did your understanding of perseverance change during class today?
I got really frustrated but I calmed down and got it.

(c)

Fig. 5 These teaching tips include questions that foster perseverance.

For choosing great tasks:
- Does the problem lead to several methods of solution?
- Is it challenging but achievable for your students?
- Does it offer opportunities for rich mathematical discourse, reflection, and extension?
- Does it build student understanding on significant mathematics content or doing mathematics?

For making strategies explicit for problem solving:
- What strategy are you using?
- Work on the problem again; use others' strategies, and see if that will help you.

For demarcating phases in the problem-solving process:
- Share with someone near you something you discovered while you were working that really helped you and that you would use again.
- Describe one thing to your partner but explain your thinking.
- Pair with someone who has a different strategy than yours and confer.
- What did you do when you got stuck?
- Where are you [in the process] right now?

For naming feelings attendant to problem solving:
- How many of you got frustrated? What did you do about that?
- How many of you had that aha! moment and changed as a result of that conversation?

For narrating internal processes:
- What made you change your mind?
- Tell me something you learned from talking with people.

year, Drauch would cue students to reference the SMPs as they worked on various problems; by this time, later in the school year, students did so without prompting.

Later, after students were directed to pair with someone who worked on the problem in a different way, the teacher gave voice to the feeling of discovery that comes during problem solving:

> How many of you had that aha! moment and changed as a result of that conversation?

5. Narrating internal processes

Related to naming feelings attendant to problem solving, the teacher narrated the internal processes in problem solving, which we think helps students self-regulate and stick with what can otherwise feel like an inefficient or unproductive use of time. The last category, naming feelings, concerns students' affective states. This category addresses the cognitive work of problem solving and helps students develop an awareness of that cognitive work by giving it voice. For example, the teacher asked, "What made you change your mind? Tell me something you learned from talking to people." These prompts legitimate the internal processes that students experience when they work on mathematical problems and reinforce norms that it is OK to try something, abandon it, and then try something else, and it is expected that students learn from collaborating with others.

FOSTERING PERSEVERANCE

The teachers' goal in this lesson was direct work on fostering perseverance. Accordingly, the script that teachers designed had no references to the content of the problem at all. Instead, the teacher talk focused on the metacognitive work involved for students to find solutions themselves and persevere in doing so. Class time was devoted to thinking and processing, and the teacher's role was to create and maintain the space for this. By the end of the lesson, all students had arrived at a solution, although they left class still debating between two variants that had been proposed. Because mathematical content played a secondary role, the exit ticket that teachers designed addressed facets of students' perseverance.

Figure 4 shows a number of students' exit tickets that were handed out at the end of the lesson. We see students reflecting on feelings and how group work helped students persevere. These student comments link directly to the categories of teacher moves for fostering perseverance that we articulated earlier: making task selections, offering varied participation structures, making strategies explicit, giving voice to affective states, and narrating internal processes. Students were able to name specific problem-solving strategies ("I looked back and started all over, and then I got my answer"); the social exchanges occasioned by the varied participation structures were helpful to some students ("My friends wouldn't let me give up"). The attention to affective states allowed others to keep at it ("I got really frustrated, but I calmed down and got it"). We can imagine using a version of this lesson early in the school year as part of an effort to establish a classroom culture that values productive struggle, and using the questions in the Teaching Tips (see **fig. 5**) to sustain these norms throughout the school year.

The widespread consensus is that making sense of problems and persevering in solving them (CCSSI 2010, p. 6) is crucial for students' proficiency in mathematics. But this disposition does not take root and grow on its own. Teachers themselves have to persevere in enacting instructional moves that cultivate perseverance in their students.

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