KATM Bulletin

Kansas Association of Teachers of Mathematics

February 2015

Friday, October 16th

Maize High School

KATM Fall Conference

Inside this issue:

A Message from our President

I write my last letter as President of KATM at time of uncertainty in Kansas. There are many things educators are anxiously waiting to find out more about in the upcoming year. School funding, new assessments, and teacher evaluations are on all of our minds. All of these topics are important to teachers. Despite these unknowns, we continue our focus on our students and their math education.

With funding at a cross roads, it is important to find alternative ways to continue professional development. Belonging to KATM and reading through this Bulletin would be a great way to learn from other teachers, while seeing what they are doing in their classrooms around the state. Recently, our editor has received permission to reprint outstanding articles from NCTM, which is a great resource to you as well. We have also started to make sure that each issue has something for elementary, middle and high school level mathematics. Another opportunity for professional development is our Fall Conference that will be in Maize, near Wichita, on October 16, 2015. We encourage you to attend! Lastly, we also want to remind you that we have many resources posted on our website, katm.org. You will also find more information on our conference soon so we encouraged to keep checking our site frequently.

Testing season is just around the corner. Last year, CETE began to change our assessments to align with the Kansas College and Career Ready Standards. This year, they have started the next phase, which will include performance assessments. There is an article in this bulletin that will give teachers more information about the performance assessments from Melissa Fast at KSDE. I encourage you to read through that article for some great information. Just as we ask students to preserve through problem solving, we, as educators, will need to do the same as CETE finishes up transitions to the new assessment. So, continued focusing on the 8 Mathematical Practice Standards will help prepare all students for the performance assessments as well as helping them prepare to become college and career ready.

It has been a pleasure serving as your President of KATM this year. I have been amazed and humbled by all of the outstanding math educators I have had the pleasure of meeting and working with since serving on the board. Becoming active in KATM has definitely helped me be a better math teacher and reignited my passion for teaching Math! I encourage you to get involved in KATM and promote KATM to your colleagues, so we all can continue to learn from each other!

Stacey Bell

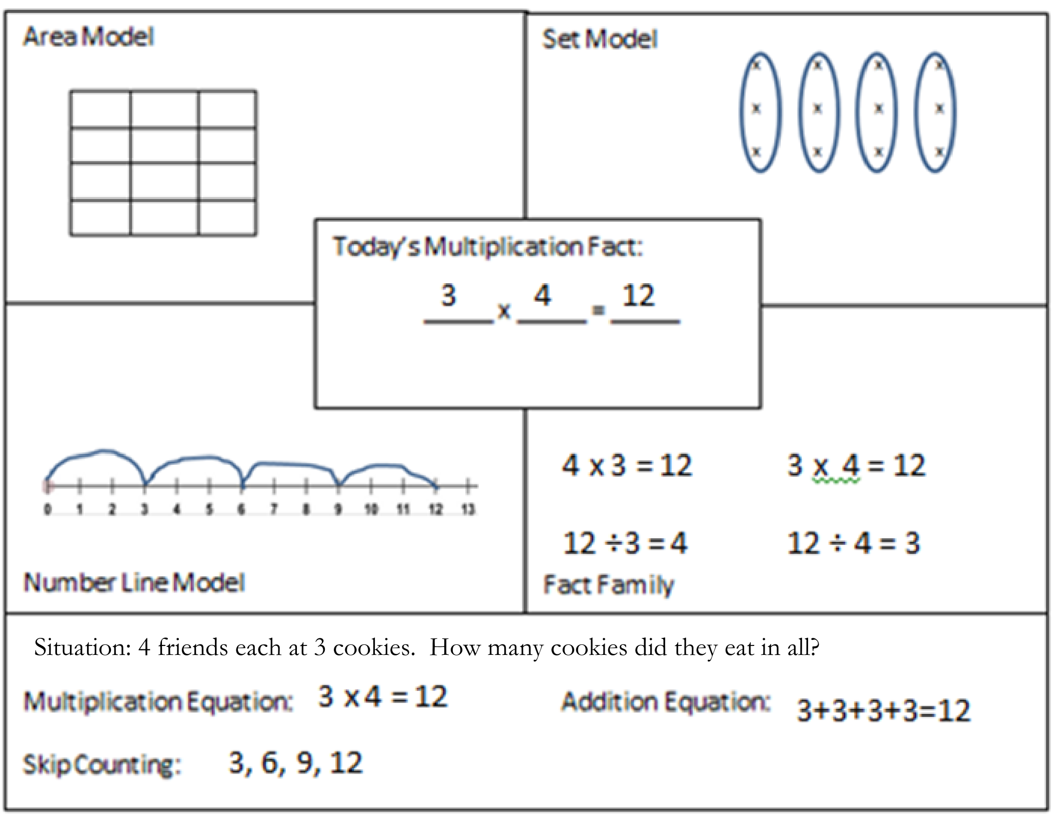
Stacey Bell
President, KATM
president@katm.org

Focus Issue: SMP #4 Model with Mathematics

CCSM Math Practice #4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

(from corestandards.org)



An example of modeling in the elementary grades, giving students a chance to make connections between multiple representations.

Model with Mathematics

For elementary students, this includes the contextual situations they encounter in the classroom. When elementary students are first studying an operation such as addition, they might arrange counters to solve problems such as this one: there are seven animals in the yard, some are dogs and some are cats, how many of each could there be? They are using the counters to model the mathematical elements of the contextual problem—that they can split a set of 7 into a set of 3 and a set of 4. When they learn how to write their actions with the counters in an equation, 4 ⍰ 3 □ 7, they are modeling the situation with numbers and symbols. Similarly, when students encounter situations such as sharing a pan of cornbread among 6 people, they might first show how to divide the cornbread into 6 equal pieces using a picture of a rectangle. The rectangle divided into 6 equal pieces is a model of the essential mathematical elements of the situation. When the students learn to write the name of each piece in relation to the whole pan as 1/6, they are now modeling the situation with mathematical notation.

When given a problem in a contextual situation, mathematically proficient students at the elementary grades can identify the mathematical elements of a situation and create a mathematical model that shows those mathematical elements and relationships among them. The mathematical model might be represented in one or more of the following ways: numbers and symbols, geometric figures, pictures or physical objects used to abstract the mathematical elements of the situation, or a mathematical diagram such as a number line, a table, or a graph, or students might use more than one of these to help them interpret the situation. For example, when students are first studying an operation such as addition, they might arrange counters to solve problems such as this one: there are seven animals in the yard, some are dogs and some are cats, how many of each could there be? They are using the counters to model the mathematical elements of the contextual problem—that they can split a set of 7 into a set of 3 and a set of 4. When they learn how to write their actions with the counters in an equation, $4 + 3 = 7$, they are modeling the situation with numbers and symbols. Similarly, when students encounter situations such as sharing a pan of cornbread among 6 people, they might first show how to divide the cornbread into 6 equal pieces using a picture of a rectangle. The rectangle divided into 6 equal pieces is a model of the essential mathematical elements of the situation. When the students learn to write the name of each piece in relation to the whole pan as 1/6, they are now modeling the situation with mathematical notation. Mathematically proficient students are able to identify important quantities in a contextual situation and use mathematical models to show the relationships of those quantities, particularly in multistep problems or problems involving more than one variable. For example, if there is a Penny Jar that starts with 3 pennies in the jar, and 4 pennies are added each day, students might use a table to model the relationship between number of days and number of pennies in the jar. They can then use the model to determine how many pennies are in the jar after 10 days, which in turn helps them model the situation with the expression, 4 □ 10 ⍰ 3. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. As students model situations with mathematics, they are choosing tools appropriately (MP.5). As they decontextualize the situation and represent it mathematically, they are also reasoning abstractly (MP.2).

Middle School

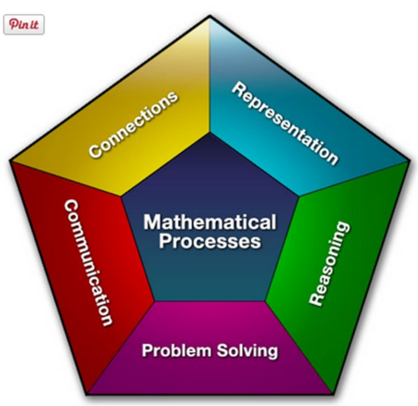
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. This might be as simple as translating a verbal description to a mathematical expression. It might also entail applying proportional reasoning to plan a school event or using a set of linear inequalities to analyze a problem in the community. Mathematically proficient students are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. For example, they can roughly fit a line to a scatter plot to make predictions and gather experimental data to approximate a probability. They can recognize the limitations of linear models in certain situations, such as representing the amounts of stretch in a bungee cord for people of different weights. They are able to identify important quantities in a given relationship such as rates of change and represent situations using such tools as diagrams, tables, graphs, flowcharts and formulas. They can analyze their representations mathematically, use the results in the context of the situation, and then reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

From Elaborations *Draft, 12 February 2014, comment at commoncoretools.wordpress.com.*

Greetings Kansas Math Teachers—

I am so excited about this issue of our KATM Bulletin! In this issue, we continue with our in-depth exploration of the Standards for Mathematical Practice. This month, model with mathematics (SMP #4) is the focus. We are so glad to share lots of great classroom tips, lessons and resources. We are also so excited to share several articles from NCTM publications. We hope the addition of these awesome articles will make our Bulletin an even greater resource for Kansas teachers. These additions have made this Bulletin VERY long—please refer to the table of contents to find the information that is targeted to your grade level.

We'd also like you to have some fun as you peruse this issue. In this issue, I have hidden the symbol below on several different pages. Your job is to find this symbol on each of the pages. E-mail me the sum of the page numbers that contain this symbol (this page doesn't count). My e-mail address is wilcojen@usd437.net. All correct entries will be entered in a drawing for a $10 Amazon gift card!



KATM is also excited to announce a new way for your to extend your KATM membership for FREE! If you refer 3 new members who become KATM members, your membership will be extended for another year at no cost to you. So tell all your friends about why you're a member of KATM—it's a win-win situation.

Happy reading!

Jenny Wilcox

What does this look like in my classroom?

Practice #4 Model with mathematics

Students:	Teachers:	Question to Develop Mathematical Thinking
• Analyze and model relationships mathematically (expression or equation) • Represent mathematics to describe a situation either with an equation, diagram and interpret the results to draw conclusion • Apply the mathematics they know to solve everyday problems • Can determine if a model makes sense and/or needs modified	• Allow time for the process to take place (model, make graphs, write equations to describe a situation, etc.) • Provide meaningful, real world, authentic, performance-based tasks (non traditional problems) • Provides contexts for students to apply mathematics learned	• What number model could you construct to represent the problem? • What are some ways to represent the quantities? • What's an equation or expression that matches the diagram..., number line.., chart..., table..? • Where did you see one of the quantities in the task in your equation or expression? • Would it help to create a diagram, graph, table...? • What are some ways to visually represent...? • What formula might apply in this situation?

Implementation Characteristics: What does it look like in planning and delivery?

Task: elements to keep in mind when determining learning experiences

☐ Is structured so that students represent the problem situation and their solution symbolically, graphically, and/or pictorially (may include technological tools) appropriate to the context of the problem.

☐ Invites students to create a context (real-world situation) that explains numerical/symbolic representations.

☐ Asks students to take complex mathematics and make it simpler by creating a model that will represent the relationship between the quantities.

☐ Requires students to identify variables, compute and interpret results, report findings, and justify the reasonableness of their results and procedures within context of the task.

Teacher: actions that further the development of math practices within their students

☐ Demonstrates and provides students experiences with the use of various mathematical models.

☐ Questions students to justify their choice of model and the thinking behind the model.

☐ Asks students about the appropriateness of the model chosen.

☐ Assists students in seeing and making connections among models.

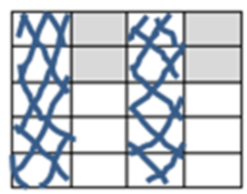
☐ Give students opportunity to evaluate the appropriateness of the model.

Charts provided courtesy of *Implementing Standards for Mathematics (SMP) "Look/Listen For" by Melisa Hancock – 2012 (Questions by Learning Services USD 259) and Learning Services PreK-12 document from Melissa Fast of KSDE*

How can I help my students model mathematically?

Math Modeling

Concept: Part-to-Part Ratio

I can write it with numbers. 5:2 5 to 2 $\frac{5}{2}$	I can draw a picture of it. 
I can write it with words (or a story problem). *This means there are 5 of one thing for every 2 of another thing. For example, if I have 5 triangles, I need 2 stars. But if I have 10 triangles (2 sets of 5) I need 4 stars (2 sets of 2).*	I can model it using *graph paper* math tools and explain my thinking.  *I made lines on 5 squares for every 2 squares that I shaded.*

Latasha bought a concert ticket. She does not remember the price of the ticket, but she knows she had to pay $1 in tax. Sales tax is 8%. She drew a percent bar to find the original price. Now that she knows that 1% is 0.125, she can multiply by 100 to get to 100%. The ticket must have cost $12.50.

Another method to model percent is with a percent table. Below is Latasha's problem modeled on a percent table.

Percent	8% (tax)	1%	100% (total ticket price)	108% (total price with tax)
Dollars	1	0.125	12.50	13.50

Model with Mathematics through the grades….connections to classroom practices

1st grade

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation... They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Liz O'Neill works with her first grade students (http://www.insidemathematics.org/classroom-videos/formative-re-engaging-lessons/1st-grade-math-base-ten-menu/lesson-part-2) engaging them in composing and decomposing numbers within twenty. Her students play a game called "How Many are Hiding?" Pairs are given a bag with 10 cubes, a paper plate, and the "How Many Are Hiding Recording Sheet." One partner takes some of the cubes and "hides" them under the plate. The remaining are placed on the top. The second partner uses sentence frames to answer the questions "What number do you see?", "How many are hiding?", "How do you know __ are hiding"? In addition, the answers are recorded. Roles are then reversed. The partner game gives students practice in composing and decomposing numbers within ten. Students who have completed the original task are challenged with changing the total number of cubes to 15 or 20.

5th grade

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation... They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense…

In a packet of mixed candies there are 2 fruit centers for every 3 caramel centers. There are 30 candies in the packet. How many caramel centers are there?

Hillary Lewis-Wolfsen invites students to examine the above problem about proportions and ratios with a strategy used by a student to organize the information in the problem. In this clip (http://www.insidemathematics.org/classroom-videos/public-lessons/5th-grade-math-proportions-ratios/problem-3-part-a) she gives students “private think time” to work the problem again, refreshing their memories about the problem, asks them to use their “think sheets” to record their ideas, then has them turn to a partner and share and defend their thinking. In the discussion, the pairs share where they think the example student’s strategy reflected misunderstandings of the quantities.

7th/8th grade

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. They are able to identify important quantities in a practical situation and map their relationships using such tools as ...tables.... They can analyze those relationships mathematically to draw conclusions.

Cecilio Dimas leads a lesson on making comparisons between three different financial plans, helping students use multiple representations of mathematical problems: verbal, tabular, graphical, and algebraic generalization. In this clip (http://www.insidemathematics.org/classroom-videos/public-lessons/7th-8th-grade-math-comparing-linear-functions/introduction), Dimas connects to the prior day's lesson, in which the class "started a conversation about the economic status of our world [and] about making responsible decisions when we're spending our money." His students share that they were to represent various DVD rental plans using verbal and tabular representations.

# of Movies	MB	OF	MF
0	0	12	18
1	3	25	18
2	6	38	18

9th/10th grade

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation... They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense...

Cathy Humphreys leads an extended exploration of a proof of the properties of quadrilaterals, helping students learn to investigate, formulate, conjecture, justify, and ultimately prove mathematical theorems. In this clip (http://www.insidemathematics.org/classroom-videos/public-lessons/9th-10th-grade-math-properties-of-quadrilaterals/tuesday-introduction-part-a), Humphreys introduces the task by posing a problem as a real-life investigation in which a kite manufacturer who "only manufactures quadrilateral kites", and needs to know the properties of convex quadrilaterals that will always result in a given kite shape, saying "how to do the sticks is the issue." The students work in groups to give prototype advice to this manufacturer, so that any time an order comes in, the manufacturer will always know "what kind of sticks to put in the kite and how they are to be put together."

All excerpts from insidemathematics.org. Video clips and lesson materials available.

KATM Cecile Beougher Scholarship
ONLY FOR ELEMENTARY TEACHERS!!

A scholarship in memory of Cecile Beougher will to be awarded to a practicing Kansas elementary (K-6) teacher for professional development in mathematics, mathematics education, and/or mathematics materials needed in the classroom. This could include attendance at a local, regional, national, state, or online conference/workshop; enrollment fees for course work, and/or math related classroom materials/supplies.

The value of the scholarship upon selection is up to $1000:

- To defray the costs of registration fees, substitute costs, tuition, books etc.,
- For reimbursement of purchase of mathematics materials/supplies for the classroom

An itemized request for funds is required. (for clarity)

REQUIREMENTS:

The successful candidate will meet the following criteria:

- Have a continuing contract for the next school year as a practicing Kansas elementary (K-6) teacher.
- Current member of KATM (if you are not a member, you may join by going to www.katm.org. The cost of a one-year membership is $15)

APPLICATION:

To be considered for this scholarship, the applicant needs to submit the following no later than June 1 of the current year:

1. A letter from the applicant addressing the following: a reflection on how the conference, workshop, or course will help your teaching, being specific about the when and what of the session, and how you plan to promote mathematics in the future.
2. Two letters of recommendation/support (one from an administrator and one from a colleague).
3. A budget outline of how the scholarship money will be spent.

Notification of status of the scholarship will be made by July 15 of the current year. Please plan to attend the KATM annual conference to receive your scholarship. Also, please plan to participate in the conference.

SUBMIT MATERIALS TO:

Betsy Wiens
2201 SE 53rd Street
Topeka, Kansas 66609

Go to www.katm.org for more guidance on this scholarship

KATM Cecile Beougher Scholarship Use

It was my honor to receive the Cecile Beugher Scholarship this past fall at the KATM conference. As we all know, money to use in classrooms is not often available to classroom teachers, but this scholarship allowed me to reflect on my math instruction time and purchase items to make our time even more meaningful. I chose to use this scholarship to purchase a Kim Sutton book, to increase the technology available to my students not only during our math time, but throughout the day and to further myself professionally. The two tablet devices and the book I chose to purchase allowed me to rethink the time after a math lesson that can often be unproductive as I'm helping some students and others are finishing their work quickly. Now, after completing their daily work, students are engaged in math until math time is over either through the devices, math games (found in Kim Sutton's book) or other activities such as 100 piece puzzles donated by parents. Students enjoy knowing exactly what is expected of them and I have been impressed with how much better our time is spent now with just a few meaningful tweaks. The remainder of my Scholarship I chose to use toward the registration to begin my National Board Certification. I believe this will allow me to be a more reflective teacher and be able to provide an even better learning environment for my students. Even though this will be a three year process (due to the new National Board changes) I am thankful for the ability to jump into this learning opportunity! Thank you to KATM for the ability to not only better my classroom, but also myself. Amy Johnston

Capitol Federal Mathematics Teaching Enhancement Scholarship

Capitol Federal Savings and the Kansas Association of Teachers of Mathematics (KATM) have established a scholarship to be awarded to a practicing Kansas (K-12) teacher for the best mathematics teaching enhancement proposal. The scholarship is $1000 to be awarded at the annual KATM conference. The scholarship is competitive with the winning proposal determined by the Executive Council of KATM.

PROPOSAL GUIDELINES:

The winning proposal will be the best plan submitted involving the enhancement of mathematics teaching. Proposals may include, but are not limited to, continuing mathematics education, conference or workshop attendance, or any other improvement of mathematics teaching opportunity. The 1-2 page typed proposal should include

- A complete description of the mathematics teaching opportunity you plan to embark upon.
- An outline of how the funds will be used.

An explanation of how this opportunity will enhance your teaching of mathematics.

REQUIREMENTS:

The successful applicant will meet the following criteria:

- Have a continuing contract for the next school year in a Kansas school.
- Teach mathematics during the current year.

Be present to accept the award at the annual KATM Conference.

APPLICATION:

To be considered for this scholarship, the applicant needs to submit the following no later than **June 1 of the current year.**

- A 1-2 page proposal as described above.

Two letters of recommendation, one from an administrator and one from a teaching colleague.

PLEASE SUBMIT MATERIALS TO:

Betsy Wiens, Phone: (785) 862-9433, 2201 SE 53rd Street, Topeka, Kansas, 66609

KATM Spring Election Nominations

David Fernkopf, President-Elect

My name is David Fernkopf and I have been a member of KATM for over six years. I am currently the KATM Treasurer and was Zone 2 rep for five years. I am an elementary principal at Overbrook Attendance Center part of Santa Fe Trail USD 434. I feel that the work KATM does for teachers across the state is very important, and I would like to help with this work, by being a future leader for KATM.

Liz Peyser, VP Middle School

Middle school is squarely in the middle and the new standards have placed an increased importance on 6-8 learning! Middle school teachers need to know it all - what the K-8 standards are doing to prepare students and where the 9-12 standards take them. I have been involved in all aspects during the transition to new standards and will continue to provide guidance to middle school math teachers as Vice President of Middle Schools.

As a curriculum coach I research current issues around middle school math and offer responses at the district level. I have also worked closely with KSDE as a trainer-of-trainers for the KSDE math academies to provide professional development for the math standards to Kansas school districts. As the current Vice Principal for Middle School I bring this expertise to KATM to communicate information to all middle school math teachers across the state. I have worked closely with the editor to enhance the quarterly bulletin for readers. Our goal is for the bulletin to provide guidance in teaching math concepts and for understanding the Math Practices. This year each issue has a Math Practice focus, with all articles supporting understanding of that Practice. Using research, I contribute articles to this quarterly journal that focus on conceptual understanding of mathematics and how the standards work in learning sequences from K-8 or 6-12. I have also provided leadership around the contentious "acceleration" issue by providing an explanation and solutions to how the new standards will impact current middle school practices. With your support, I hope to be able to continue this work on behalf of Kansas math teachers.

Stacie Stricker, VP Middle School

My name is Stacie Stricker and I have taught for 21 years. I have taught the last 8 years at Maize South Middle School teaching Pre-Algebra and math Improvement. I have also taught special in education in Olathe and Shawnee Mission as well as fifth grade in Blue Valley. I currently teach adjunct classes at Friends University helping teachers incorporate activities and common core into their math classrooms.

I am excited to run for the position of Vice President of Middle School because I have enjoyed learning the middle school math curriculum, as well as, implementing it into the classroom; now I would like to collaborate with others throughout the state.

I have one son that attends KU.

KATM Spring Election Nominations

Jerry Braun, VP College

My name is Jerry Braun. I am running for the position of Vice President for Colleges on the KATM Board. Mathematics education has been my passion for many years. I received my BA in Mathematics Education from Fort Hays State University in 1995 and my MS in Instructional Technology from Fort Hays State University in 2007. I have 15 years experience as a middle school mathematics teacher, 2 years experience as an Education Consultant for Southwest Plains Regional Service Center, 3 years experience as a k-12 instructional math coach and 7 years experience teaching online education and math courses for Fort Hays State University as an adjunct instructor. I have also filed to run for a position on the USD489 Board of Education. I have been a previous KATM board member serving as Zone 1 Representative, Vice President for Middle School and KATM President. Through this affiliation with KATM, I also served on the Common Core Standards Review committee, served twice as KATM conference chair/co-chair, and presented at numerous conference, KSDE summer academies and other training opportunities. I would like to rejoin the KATM board to help support mathematics instruction and preparation of mathematics teachers as well as math education in general. I look forward to hopefully serving you as a member of the KATM Board.

KATM January Board Meeting Highlights

- Webmaster replacement discussion and process for transition
- Request for funds:
  - KEEN Conference - $250 for snacks
  - Learning Forward Kansas – Door prizes provided
  - NCTM Subscription for Editor of the Bulletin
- Ideas from board about how to increase membership – list benefits of membership on the website, board member/zone reps need to bring in a certain number of memberships per year, passwords to receive a copy of the bulletin from the website, online payment
- KLFA Update
- KSDE Update
  - Cut scores will be established Summer of 2015 and released during Winter 2015, no scores for Performance Assessments
- Scholarships – Betsy
  - No scholarship submissions yet
  - Idea of using social media to spread the word about the scholarship opportunity
- Membership committee will add a referred by line to the membership form and provide members with a free year of membership if they refer 3 members to join.

Information about the Performance Task, KSDE Update, Melissa Fast

I've been receiving many questions regarding the performance tasks which will be on the state assessment this year and wanted to send out the information I have at this time. Below is a summary of the details I have at this time, as I become aware of further information I will pass it along to the field as well as post it on my assessment page at: http://community.ksde.org/Default.aspx?tabid=5418

- **What is a performance task?**

Performance tasks are extended activities that measure a student's ability to integrate knowledge and skills across multiple standards—a key component of college and career readiness. Performance tasks will be used to better measure capacities such as depth of understanding, and complex analysis, which cannot be adequately assessed with selected- or constructed-response items. Some parts of the performance tasks can be scored automatically; many parts will be hand-scored by professionally trained scorers.

- **How long will the performance task section of the state assessment take my students?**

We anticipate that the tasks should take students between 15-60 minutes, with the lower grades probably taking closer to 15-30 min and the upper grades taking up to 60 min. I'm sorry we can't offer any more specific numbers, but the timing of the performance tasks is something we will find out through the field test.

- **Will there be sample tasks available to the field prior to the assessment window this year?**

Unfortunately there will not be any performance task samples to offer the field prior to the 2015 assessment window as the items need to be field tested prior to being released. We will be able to offer sample tasks in future years.

- **Will there be a separate script which will need to be read during the performance task section of the state assessment?**

Yes, we will release more information with the scripts specifically written for the performance task section prior to the assessment window and an addendum will be added to the assessment manual.

- **Who will score and how will the performance tasks be scored?**

There are still a lot of details which need to be worked out in the area but what I do know at this time is a distributed scoring model will be utilized, all scoring will be online, scoring will be completed prior to the start of the 2015-2016 school year, and it is likely that most teachers will be involved in the scoring process.

- **Who will take the performance task section of the math assessment?**

For the 2015 assessment grades 3-8 will have 1 performance task to complete on the assessment. With the change from 11th grade to 10thgrade assessment there will not be a performance task section in high school for the 2015 assessment window, but the 2016 assessment will have a performance task section at the high school level as well.

- **How will the performance task section be administered on the assessment?**

The performance task section will be administered in KITE just like the other sections of the assessment.

- **Will there be a separate testing ticket for the performance task section of the assessment?**

Yes there will be a total of 5 testing tickets for the math assessment: 1 for session 1 which will consist of 25 questions taken by every student, 3 for the 15-question sections of the assessment, and 1 for the performance task section.

- **Does the performance task section need to be administered only after all the other sections are completed?**

No once the testing window is open (March 9, 2015 to May 15, 2015) students may take the performance task section at any point in their assessment process.

KLFA

Commissioner Watson Makes KLFA Visit

January 8, 2015

Incoming Commissioner Dr. Randy Watson shared details about the upcoming "listening" tour he and state board members will hold in January and February at the January 8 Kansas Learning First Alliance meeting held at the KNEA building. He will introduce the tour at the KASB Governmental Relations Network Seminar Jan. 22 with an abbreviated version he hopes will attract legislators. Other cities on the tour include Arkansas City, Wichita, Salina, Hutchinson, Hays, Colby, Sublette, Greenbush, Emporia, Topeka, Kansas City, Olathe and Hiawatha. He also discussed his plans to make education a student-focused endeavor to which the system responds, focusing on STUDENT SUCCESS! His presentation concluded with a discussion on building communications channels to share important information about schools, their achievements and challenges.

Dr. Doug Moeckel, KASB Deputy Executive Director, led a session on change and leadership that dovetailed well with Dr. Watson's presentation. Dr. Moeckel used the material he has shared frequently with boards and administrators.

As part of a year-long study of change and the Standards of Professional Learning, **Dayna Richardson**, KLFA chair, focused on the standard **Learning Designs**. Strategies/actions that have the biggest impact on classroom practices and student success were shared.

Finally, the meeting celebrated the 16th anniversary of the organization, which is modeled after the Learning First Alliance. Twenty-five members representing 18 organizations attended the meeting. Those 18 organizations unanimously approved the **Association of Teacher Educators-Kansas (ATEK)** to membership in KLFA. The next meeting will be April 16 at KASB.

For more information on KLFA, visit its Website at KLFA.org.

Have Your Pi and Eat it Too!

This is the second in a series of Pi Day articles.

Dr. Janet Stramel

Pi Day – March 14

Yes, Pi Day happens every year. But this year, Pi Day is truly worth celebrating: a once-in-a-century experience. 3.141592653 (March 14, 2015 at 9:36:53 a.m.) will be the most places of π we will have on Pi Day till the next century.

Most people have heard of pi, and know that it has something to do with a circle. It is the ratio of a circle's circumference to its diameter, and represented by the Greek letter "π." No matter how big a circle is, the value of π is always the same, 3.1415926… But did you know that Pi Day is an official day? According to House Resolution 224 of the first session of the 111th Congress of the United States, "Whereas the Greek letter (Pi) is the symbol for the ratio of the circumference of a circle to its diameter," resolves to support the designation of "Pi Day" to encourage "schools and educators to observe the day with appropriate activities that teach students about Pi and engage them about the study of mathematics" (2009, McCullagh).

Common Core Standard Connections

In the Kansas Core Standards, seventh grade students "know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle," (7.G.4) and "solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms" (7.G.6).

But for younger students, you can introduce ideas such as size, shape, and circumference as well as in the kitchen to explore circumference, diameter, and fractions by making pizza pies.

Websites for Activities to Help Celebrate Pi Day!

There are many websites with activities to help you celebrate Pi Day. Here are just a few:

http://www.nctm.org/resources/content.aspx?id=2147483830

http://www.piday.org/2008/2008-pi-day-activities-for-teachers/

Finding the relationship between the circumference and diameter of circles

Submitted by Liz Peyser, Wichita Public Schools

Objective: Students understand and verbalize that it takes about three diameters to go around a circle. Students create a rule to show this relationship.

CCSS Standards for Mathematical Practice connections: #3, #4, #7 and #8.

CCSS domain connections: Expressions and Equations, Functions, Geometry

Materials (for groups or partners):

- Tape measures, one per group
- 5 – 7 identical circles, numbered #1, one per group
- 5 – 7 identical circles of a different size, numbered #2, one per group
- 5 – 7 identical circles of a third size, numbered #3, one per group
- Additional circles for more data collections
- Graph paper, or a laptop with LoggerPro software to collect class data of ordered pairs.
- Calculators

Opening (5-7 minutes): On a chart draw a large circle. Explain two vocabulary words: circumference (distance around the circle) and diameter (distance across the circle through the center). Optional: introduce the word radius if this has not been introduced yet. Demonstrate how to measure the circumference and diameter with the tape measure (measuring could also be practiced the day before this lesson). Explain how to record measurements in the chart

Worktime #1 (5 to 10 minutes): Each group should have three circles (#1, #2, #3) and a tape measure. Have students measure the three circles and complete the chart. Monitor students as they complete the charts to make sure measurements are reasonable. Return items to counter.

Closing: Before students go on to part B, collect some information from the students to complete a chart together to make sure that their circumference measurements are equal to about 3 diameters and have a brief conversation of what pattern they are seeing. This is just to ensure that measurements are reasonable and students can check if they have unreasonable measurements.

Worktime #2 (15 minutes): complete the rest of the activity sheet

Closing (10-15 minutes): Students should be able to demonstrate, explain, verbalize the rules that show the relationship between the circumference and diameter. **Students should understand that it takes about 3 diameters to go around a circle**. If the students or teacher want to discuss "pi" it should be in the context that the number represents how many diameters are needed to create a circumference. The slope of the line will show the 3 to 1 relationship as a slope of about 3. Collect rules on an artifact. Note-taking is at the end.

Extension questions: application of rules that are created

A. Measure the circumference and diameter of each object and fill in the chart

Circle number	Diameter (X)	Circumference (Y)	Ordered pair (x, y)	About how many diameters are needed to complete a circumference? (Estimate)	Ratio: Circumference / diameter Convert to a decimal
1.					
2.					
3.					

B. What is the pattern that you notice in your estimations and your exact answers in Chart A?

C. Estimate how many diameters it takes to complete a circumference?

D. Show this with a picture.

E. Complete this chart by estimating:

Diameter	
1	
	15
	36
4	

F. What did you do to find your answers for the missing spaces?

G. It takes a little more than three diameters to complete a circumference. It takes about 3.14 diameters. Complete the chart again, using this new information

H. Write a rule for finding the diameter if you know the circumference.

I. Write a rule for finding the circumference if you know the diameter.

J. Collect data from more circles and graph the ordered pairs. Find the slope of a line of best fit. How does the slope compare to the ratios? What does the slope of the line tell you about the relationship of the circumference to the diameter?

K. Using the slope, write an equation in slope-intercept form to show the relationship:

Extension:

How could you measure the diameter of a tree at the ground level without cutting it down to see the stump?

How could you find the radius of a swimming pool without getting wet?

In the coming issues—

- *April 2015 Bulletin will focus on #3,* ***Construct viable arguments and critique the reasoning of others.***

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. (description courtesy of corestandards.org)

October 2015—Attend to precision

*December 2015—Make sense of problems and persevere in solving them

*February 2016—Reason abstractly and quantitatively

*April 2016—Look for and make use of structure AND Look for and express regularity in reasoning

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◊ Lesson plans
◊ Classroom management tips
◊ Books reviews
◊ Classroom games
◊ Reviews of recently adopted resources
◊ Good problems for classroom use
◊

Email your submissions to our Bulletin editor: wilcojen@usd437.net

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Developing Quantitative Mental Imagery

from October 2012 in Teaching Children Mathematics

Joseph is a first-grade student whose teacher is working with him to better understand the manner in which he adds. Consider the following exchange around the task 11 + 4.

Teacher: So, there are eleven under here [*covering a collection of blue counters*]. And then we put four more with it [*covering a set of red counters*].

Joseph: Six … eleven [*touching the cover on top of the red counters four times in an approximation of the same spatial pattern as the concealed counters beneath*], twelve, thirteen, fourteen, [*whispering*] fifteen. Fifteen!

Moving beyond physical interactions with materials is a significant mathematical step for students that is often difficult to take. Persistent tally-mark use, for example, among older children is a testament to this challenge. For many students, shifting away from tangible tools begins a precarious journey; teachers should support it with thoughtfully tailored instruction. Indeed, this journey begins with helping students "see" math with their "mind's eye" via the construction of quantitative mental imagery.

The vignette above shows us that Joseph is working in a somewhat novel setting. The counters are still within his physical space; however, a thick, opaque screen that prevents sight or touch intentionally conceals them. Joseph appears quite able to negotiate this task, though, and uses a counting strategy to determine the sum of 11 + 4. The manner in which he appears to reconstruct the spatial pattern of the red counters suggests some connection to the hidden materials; but how, exactly, did he perform this strategy without actually touching the materials that were beneath the cover? The key to moving away from dependence on materials lies in the construction of quantitative mental imagery.

Quantitative aspect of number

Before discussing mental imagery and arithmetic strategies, a brief examination of number is necessary. Numbers as conceptual and cultural entities possess three distinct components: verbal (i.e., *four*), numeral (i.e., 4), and quantity (i.e., □ □ □ □) (Wright 1994; Wright, Martland, and Stafford 2006). Neurological confirmation of these different aspects may be found in Dehaene's (1992) triple-code model, which describes the location of neural activity as a function of number aspect. The model explains how different parts of the brain become active depending on the aspect of number with which one is working. Dissecting the different components that comprise numbers is important here, as the remainder of this article focuses primarily on work within the quantitative aspect.

Stages of early arithmetic learning

Resulting from a series of extensive teaching experiments, the Stages of Early Arithmetic Learning (SEAL) form a model for understanding how children come to understand quantity (Steffe 1992; Olive 2001; Wright, Martland, and Stafford 2006; Wright et al. 2006). Specifically, this model leverages varying arithmetic strategies to describe how children progress through stages (see fig. 1) of increasingly sophisticated quantitative understanding. This progression describes mathematical activity beginning with *emergent counting*—where children approximate counting but are unable to determine the numerosity of a single collection of materials—all the way to the *facile number sequence*—where children may enact multiple, different non-count-by-ones strategies to negotiate arithmetic tasks. For example, a student might accomplish 12 + 5 by "placing the 10 aside," adding 2 + 5, and then "replacing the 10" to arrive at a sum of 17.

Of particular importance are the perceptual and figurative stages, during which children move from working with quantity as a physical entity toward more abstract, mental construction of quantity. If perceptual counting is considered a direct sensory experience, then figurative counting may be thought of as one step removed from a direct sensory experience. At the figurative stage, children are no longer tied to working with items they can see or touch, but rather can begin to leverage mental replays of past sensory experiences to facilitate counting acts (Clements and Sarama 2007; Rittle-Johnson, Siegler, and Alibali 2001; Steffe 1992). These mental replays have been termed *re-presentations* (as distinct from representations) to describe how children mentally re-present or re-enact a prior sensory experience to themselves (Olive 2001; Steffe 1992).

FIGURE 1

Each new phase in the Stages of Early Arithmetic Learning (SEAL) model incorporates, rather than displaces, the knowledge and understanding of prior stages.

Stages of Early Arithmetic Learning (SEAL)
(Steffe et al. 1988; Steffe 1992; Wright et al. 2006)

Emergent
Child approximates counting activity (e.g., saying number words when asked, "How many?") but is typically unable to determine the numerosity of a collection.
What it might look like: Child is presented with a collection of twelve counters and asked how many are there. Child touches some, but not all, the counters, saying, "One, two, three, five, seven, eight, nine—nine!"
Hallmark strategy: Attempting to count a collection

Perceptual
Child can determine the numerosity of collections when physical materials are available for counting but is unable to negotiate arithmetic tasks in the absence of physical materials.
What it might look like: Child is presented with collections of nine counters and five counters and asked, "How many altogether?" The child touches each of the counters while saying, "One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen!"
Hallmark strategy: Physically interacting with materials to count collections

Figurative
Child can negotiate arithmetic tasks in the absence of physical materials by generating mental imagery of past sensory experiences referred to as *re-presentations*.
What it might look like: Child is presented with a collection of nine counters and five counters; the counters are then concealed. Child is asked how many altogether. The child looks away, begins counting at one (may or may not sequentially raise fingers), and says, "One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen—fourteen!"
Hallmark strategy: Continuing the count from one when materials are physically unavailable

Initial number sequence
Child can negotiate arithmetic tasks in the absence of physical materials by constructing a single chunk of a number, referred to as *a numerical composite*, and then counting on from this chunk.
What it might look like: Child is presented with collections of nine and five counters, and the counters are then concealed. Child is asked how many altogether. Child counts on from nine (may or may not sequentially raise fingers) and says, "Ten, eleven, twelve, thirteen, fourteen—fourteen!"
Hallmark strategy: Counting on when materials are physically unavailable

Facile number sequence
Child can negotiate arithmetic tasks in the absence of physical materials by constructing multiple chunks of numbers, referred to as *abstract numerical composites*, and decomposing/recomposing these chunks.
What it might look like: Child is presented with collections of nine and five counters, and the counters are then concealed and asked how many altogether. The child responds immediately, "Fourteen—I borrowed one from the five to make ten, and then just added ten and four in my head."
Hallmark strategy: Multiple non-count-by-ones strategies when materials are physically unavailable

Perceptual and figurative arithmetic strategies

Jennifer and William are first graders at the same school. In the exchanges that follow, a teacher attempts to determine how each child understands and works with quantity. Consider first the exchange between Jennifer and her teacher.

Teacher: Five reds [placing a cover over a collection of five red counters] and four blues [placing a cover over four blue counters]. How many altogether [waving a hand across both covers]?

Jennifer: Nine [displaying five fingers on her left hand and touching each of them with the index finger of her right hand and then displaying four fingers on her left hand and touching each of them with the index finger of her right hand].

Teacher: Let's try this one: nine red counters [placing a cover over a collection] and six blues [placing a cover over them]. How many altogether [waving her hand across both covers]?

Jennifer: Six? [*displaying five fingers on her left hand and touching each of them with the index finger of her right hand and then displaying four fingers on her left hand and touching each of them with the index finger of her right hand*]

Teacher: OK, how did you know?

Jennifer: I counted my fingers.

Jennifer is able to negotiate arithmetic tasks where the materials have been concealed; however, her strategy involves substituting the concealed materials with her own fingers. As one might suspect, when a task's sum is greater than ten, Jennifer is unable to determine the numerosity of the two collections—quite simply, she runs out of fingers. Interestingly, in similar cases, children will sometimes attempt to count other accessible, physical items (e.g., toes, bricks of a classroom wall, ceiling tiles, etc.). The key point here is that Jennifer's strategy appears perceptual; she relies on physical interaction with materials (her fingers) to operate arithmetically. In another classroom, William is working on the same quantitative tasks. Consider the following exchange between William and his teacher.

Teacher: OK, nine chips right there [*placing a cover over a collection of counters*] and six chips [*placing a cover over blue counters*] How many altogether [*waving her hand across both covers*]?

William: [*raising nine fingers sequentially and whispering*] One, two, three, four, five, six, seven, eight, nine, ten. [*Looking away from the covers and shutting his eyes*, *he lowers his fingers on one hand, raises them again sequentially, and whispers.*] Eleven, twelve, thirteen, fourteen, fifteen. Fifteen.

William has a strategy for thinking arithmetically in the absence of physical materials. Specifically, he does not need fifteen countable objects to negotiate the task (9 + 6) but can re-present involved quantities to arrive at a solution. Both Jennifer and William used their fingers during their mathematical activity; however, Jennifer appeared to consider her fingers as physical objects to be counted, whereas William raised his fingers sequentially, ostensibly to help him keep track of his *figural re-presentation*. Indeed, key to this exchange is William's capacity, via mental imagery, to move beyond physical interactions with materials in the context of arithmetic tasks. In terms of instructional next steps, Jennifer could likely capitalize on partially screened arithmetic tasks (e.g., 9 + 6, but only the second addend [6] is screened) to aid the development of her re-presentational capacity. William, already apparently re-presenting with facility, would likely benefit from tasks aimed at furthering this capacity into counting-on strategies. For example, with William, we might pose the task 32 + 3 with fully screened counters. The disparity in the two addends is designed to help William curtail his count from-one approach. The first addend is inconveniently large, and the second addend is tantalizingly small (see fig. 2).

FIGURE 2

To decide on suitable next steps, teachers need a practical way to diagnose students' mathematical understanding. These SEAL instructional examples offer such diagnostic measures.

Stages of Early Arithmetic Learning (SEAL) Instructional Examples
(Wright et al. 2006)

Emergent
Key next step: **Developing the capacity to accurately count a collection of materials**
Instructional examples: (1) Present the child with a collection of twelve counters and ask, "How many counters are in this pile?"
(2) Present the child with a collection of thirty seashells and ask, "Can you get me fourteen of those seashells?"

Perceptual
Key next step: **Developing the capacity to mentally re-enact (i.e., visualize) arithmetic experiences**
Instructional examples: (1) Present the child with a collection of nine red counters and seven blue counters. Screen the second collection and say, "Nine counters and seven more under the cover. How many altogether?"
(2) Present the child with the same task, but in this instance, screen both collections.

Figurative
Key next step: **Developing strategies to count on and count back during arithmetic tasks.**
Instructional examples: (1) Present the child with a collection of twenty-seven red counters and two blue counters. Screen both collections, and say, "Twenty-seven counters and two more under the covers. How many altogether?"
(2) Present the child with twelve counters, and then screen the collection. Remove four counters from under the screen, and place them under a second screen. Say, "We started with twelve counters, but then I took four out. How many counters are left?"

Initial number sequence
Key next step: **Developing non-count-by-ones (composite) arithmetic strategies**
Instructional examples: (1) Present the child with two 10-bundles of Popsicle sticks and four loose sticks and then screen them. Ask, "How many sticks are under the cover?" Add additional 10-bundles and single sticks, and ask, "How many now?" after each addition.
(2) Present the child with five 10-bundles of Popsicle sticks and three loose sticks, and then screen them. Ask, "How many more sticks do I need to have sixty sticks? How about to have seventy sticks?"

Facile number sequence
Key next step: **Extending composite addition and subtraction strategies**
Instructional example: Task the child with mentally solving two-digit and three-digit addition and subtraction tasks. Ask how he or she worked out the problem. Model the child's strategy with an empty number line (if the child uses a jump strategy, e.g., 36–24: "I went backwards twenty and landed at sixteen, and then four more to twelve") or a tree diagram (if the child uses a split strategy, e.g., "I took twenty from thirty and got ten. Then I took four from six and got two, so it's twelve").

Making practical diagnoses

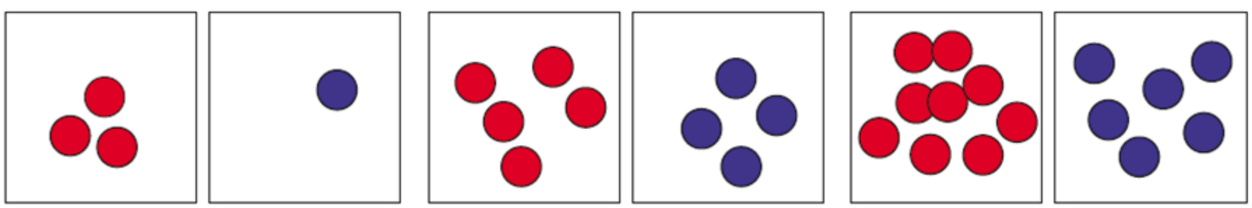
Mathematical teaching and learning is most effective when the teacher is able to enact the right task for the right child at the right time. This necessarily means that some manner of diagnosis is necessary to determine appropriate instruction. Although mathematics intervention specialists may wish to use robust, diagnostic interviews (Wright et al. 2006) to precisely ascertain students' mathematical understanding, classroom teachers often need measures that are more expedient. Ideally, cognitive determinations of a child are based on observations across a range of activities, but some relatively brief task progressions can help teachers distinguish between children's perceptual and figurative arithmetic strategies. Jennifer's and William's earlier tasks (see fig. 3) are a good way to differentiate between the strategies. As the reader observed with Jennifer, children who have yet to develop figurative arithmetic strategies will typically be unable to negotiate the third and final task in the diagnostic progression above. Thus, teachers may then tailor instruction to help these students begin to move beyond physical interactions with materials.

FIGURE 3

Students who have not yet developed figurative arithmetic tactics will typically be unable to negotiate the last step of task 2 in the diagnostic progression.

Perceptual vs. Figurative Diagnostic Task Progression

Task 1	Task 2	Task 3
3+1	5+4	9+6
Presentation:	Presentation:	Presentation:
Present three red counters, and then conceal them; present one blue counter, and then conceal it.	Present five red counters, and then conceal them; present four blue counters, and then conceal them.	Present nine red counters, and then conceal them; present six blue counters, and then conceal them.



Note: Children should not be able to see or touch counters when they are concealed.

Specific tasks to support the development of quantitative mental imagery

Returning to the notion that effective mathematical tasks are those tailored to the individual child, helping students advance their mathematical thinking and strategies beyond physical interactions requires specific instructional tactics aimed at fostering the development of mental imagery and representational capacity. Typically, this instruction features physical materials of some kind; however, to support imagery construction, the materials are often presented and then concealed. Consider this exchange with Jennifer involving random objects concealed by an opaque red screen—referred to as a linear imaging task (see fig. 4).

Teacher: Ready? [*She places the screen over a linear arrangement of four objects from left to right: a car, a bear, a rooster, and a frog.*] What's on this end [*touching the far right-hand side of the screen*]?

Jennifer: [*pausing for eight seconds*] Frog. How many things came before that frog [*dragging her finger across the screen from right to left*]? I think I put five things [*touching the screen three times in a linear pattern from right to left*].

FIGURE 4

To develop Jennifer's capacity to re-present, the teacher gave her a chance to view the materials again during the linear imaging task but replaced the screen when questioning the child.

Teacher: You think so? You want to have another look [*raising the screen*]?

Jennifer: One, two, three, four, oh.

Teacher: So [*lowering the screen*], there is a frog [*touching the far right-hand side of the screen*]. How many things came before the frog [*dragging her finger across the screen from right to left*]?

Jennifer: [*looking across the table toward a shell and a block placed in front of the teacher and touching the screen in a linear pattern from right to left*] Rooster, shell … Oh! I didn't get the shell [*motioning toward the shell in front of the teacher*]. Rooster, bear [*touching the screen two times in a linear pattern, from right to left*], car, *three* [*rapidly raising three fingers sequentially*].

Teacher: Three things came before the frog.

First, notice the manner in which the teacher allows Jennifer the opportunity to view the materials again during the task; however, the teacher replaces the screen during the questioning portions of the task. Note also the manner in which Jennifer touches the top of the screen (in a linear pattern), suggesting a productive connection between her thinking and the tool. Again, the intent with this presentation is to develop Jennifer's incipient capacity to re-present. Prior to this work, Jennifer appeared unable to move beyond perceptual arithmetic strategies—specifically those involving finger patterns.

Teachers might use many other tools to help students develop quantitative mental imagery. Consider Jennifer's work with a linear arrangement of dots—referred to as a row task—and an arrangement of animal cards (see fig. 5 and fig. 6). Similar to the linear imaging task, negotiating the row task involves re-presenting concealed objects; however, in this instance, the screened quantities are uniform in appearance and do not have any unique or distinguishing characteristics (e.g., a yellow car, a green frog, etc.). Turning to Jennifer's work with the animal cards, the use of "four-legged animals" is a fairly effective support in that the screened quantities are now grouped into more manageable and natural chunks. Also, note the manner in which this task was initially presented (as cards in a stack). After Jennifer remarked that this task is "a hard one" and that she could not work it out, the teacher adjusted the task so that Jennifer was able to physically interact with the cards, although they were facedown.

FIGURE 5

After Jennifer commented on the difficulty of this row task, the teacher micro-adjusted it so the child could physically interact with the cards, giving her confidence in her solution.

Animal Card Task

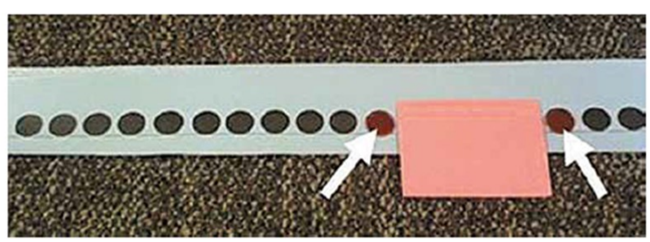
Teacher: What if I had three animal cards in a stack [*fanning a stack of three animal cards facedown and then restacking*]; how many animal legs do you think I have?
Jennifer: That's a hard one!
Teacher: Is there any way you can figure it out?
Jennifer: No, I can't do it [*shaking her head*].
Teacher: If I put them out like this [*placing the three cards in a row, facedown in front of Jennifer*] does that help?
Jennifer: [*nods*]
Teacher: How does that help you?
Jennifer: One, two, three, four [*touching the back of each card four times in the approximate location of the legs*], five, six, seven, eight, nine, ten, eleven, **twelve**.
Teacher: OK, twelve. Do you need to check it?
Jennifer: No [*shaking her head*].

FIGURE 6

This row task involves re-presenting concealed objects that are uniform in appearance and have no unique or distinguishing characteristics.

Row Task

Teacher: So let's see here; there is eleven [*placing a translucent red counter on top of the eleventh dot on the row tool*]. And I am going to cover up four [*placing a screen over the twelfth, thirteenth, fourteenth, and fifteenth dots on the row tool*]. What is this guy [*placing a translucent red counter on top of the sixteenth dot on the row tool*]?
Jennifer: Twelve, thirteen, fourteen, fifteen [*touching the screen four times in a linear pattern and audibly whispering the numbers*]. You covered up fifteen, and then that's sixteen [*touching the counter on top of the sixteenth dot on the row tool*].

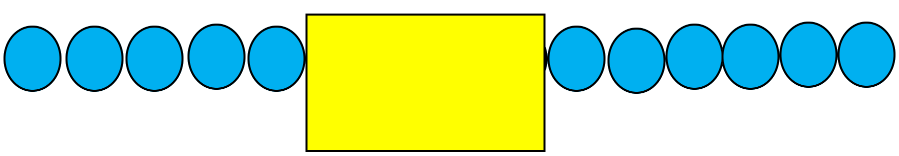


Interestingly, Jennifer did not appear to need to see the images of the animals to count the legs, but she did seem to need markers (i.e., the facedown cards) for each animal to help her keep track of her re-presentation. This modification certainly reduces the level of demand for this particular task (i.e., the introduction of perceptual markers for each animal card). However, this new task arguably presents a more meaningful and connected transition point toward work with increasingly sophisticated figurative experiences: Note the child's apparent confidence in her solution. Indeed, after several tasks where the cards are presented individually (facedown), the teacher might return to pose variations of the original task, where the cards are presented in a stack. The point is that working to help children advance their thinking and strategies in this area will often require teachers to make micro-adjustments to tools and task presentations as they are teaching, to increase task accessibility. Some potential modifications include the following:

- Increasing or decreasing the concealed quantities
- When working with multiple quantities, screening only some of them (e.g., presenting animal cards both faceup and facedown)
- Incorporating color into concealed quantities (e.g., using different color counters to denote the two addends in an addition task)
- Adding structure or patterns to concealed quantities (e.g., arranging collections of counters in domino patterns before concealing them)

Supporting imagery development

Although individual interactions may be productive means to help children develop quantitative mental imagery, such interactions are often impractical for classroom teachers; thus, teachers may elect to work with small groups of children who have all demonstrated strong perceptual counting strategies but seem unable to move beyond physical interactions with materials. Here, game contexts frequently prove useful. For example, the teacher might distribute a single six-sided dot die to each of the children and might keep two dice for herself. She rolls the two dice, announces the number, and then conceals the dice with a cover. Going around the table, each child rolls his or her die and announces the sum of the child's die plus the teacher's dice. The child with the largest sum wins the round. This game approximates partially screened arithmetic tasks that are useful in helping children transition from perceptual to figurative counting. Additionally, the use of three dice increases the probability that sums will be beyond finger-counting range.



Supporting the development of quantitative mental imagery

Adapting curriculum materials to support the development of quantitative mental imagery can be as simple as adding a cover to an existing task. For example, if the task involves children adding sections of train cars to determine the entire length of a train, one section might be inside a small tunnel obscuring two or three of the cars. With tasks involving pictorial representations of addition tasks, the picture of the smaller addend could be covered with a sticky note, preventing students from readily counting each perceptual item in the task. However, if needed, the covers can easily be removed to support the development of the quantitative mental image.

From materials to mind's eye

Given the aim for children to construct an increasingly abstract understanding of mathematics, robust support for mental imagery and re-presentation is of considerable importance. Even a brief series of tasks can afford teachers diagnostic power to design tailored and effective instruction that helps a child transition from materials to mind's eye.

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About the authors: **Jonathan N. Thomas**, **thomasj13@nku.edu**, is an assistant professor of mathematics education at Northern Kentucky University and a Kentucky Center for Mathematics faculty associate. **Pamela D. Tabor**, **pamela.tabor@hcps.org**, is a school-based elementary math specialist in Harford County, Maryland.

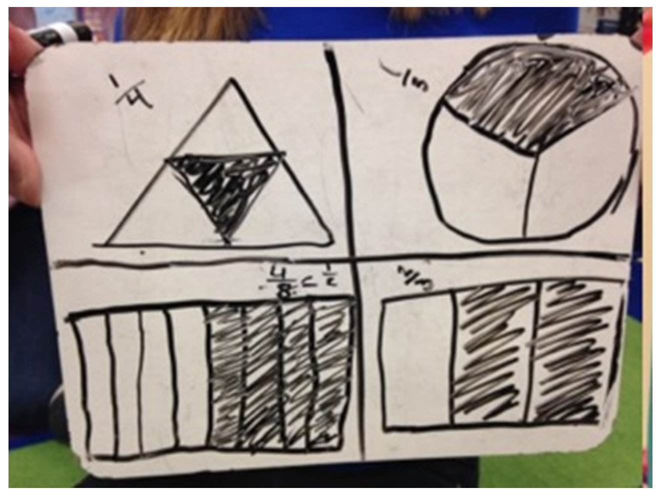
Second grade fractions conceptual lesson

Lesson developed by Erin Myers and Trisha Sharp, Shawnee Heights Elementary School

Engage

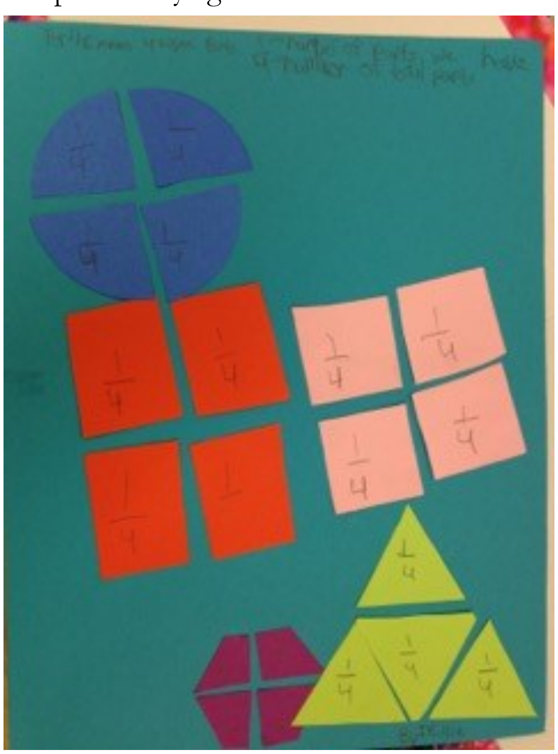
1. Begin with students with their dry erase boards. I'll ask them to divide it into HALVES. They will show me, then ask if they can do it another way. Then another (diagonal is usually last). I'll show them my board, with two parts not equal, and see if it works for "half". Lead students to realize that half must be equal parts. When do we divide things into halves in our lives? (sandwiches, oreos, …so on). Discuss the word "WHOLE" also…"So we started with a WHOLE sandwich, and divided it into HALVES?" Cooking might also come up, and demonstrate how a half cup of sugar, and another half cup of sugar makes one cup.

2. Ask them to write one-half on their board, not with a picture. Lead students to see that it can be written as words, or the fraction, and discuss that the top number is parts we HAVE, bottom number is TOTAL PARTS.



Explore

1. Students head back to their seats, and using die-cuts of different shapes (I did a square, triangle, circle, rectangle, and hexagon), they divide them into halves and glue them down to construction paper. They label each part with the proper fraction. Students start to realize that they can fold the shapes first to get more perfect halves. They also can throw away failed attempts and try again!



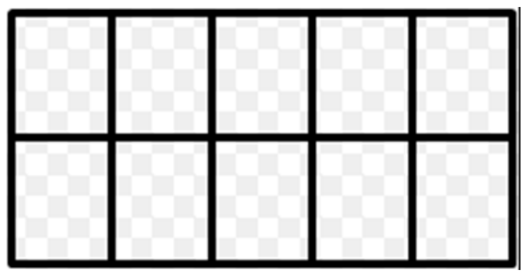
2. Repeat engage and explore with students on following days with fourths, and thirds. (Continue using measuring cups, too!) The die-cuts get fun, because some shapes are not as easy to figure out how to divide. It is exciting when one student divides an equilateral triangle into fourths correctly after

Extend

1. Write the word "fraction" on the board, and have a discussion about how it relates to what we have been doing (any part of a whole). Today is their "fraction challenge". Ask students to divide boards into fourths, and in each space ask them to draw a different shape, and a part of it shaded with the fraction to the side. Then, increase the shaded part, and ask them to change the fraction beside it. (Example." Divide a rectangle into thirds. Now shade in one part, and write the fraction beside it. Now shade in another part also. What is

the fraction now?".) Lead them to fully understand that the part shaded might change, but the total parts remains the same. This also becomes a good time for them to realize that 2 parts of 4 shaded in looks like one-half. (See picture...Not a second grade standard, but good discussion).

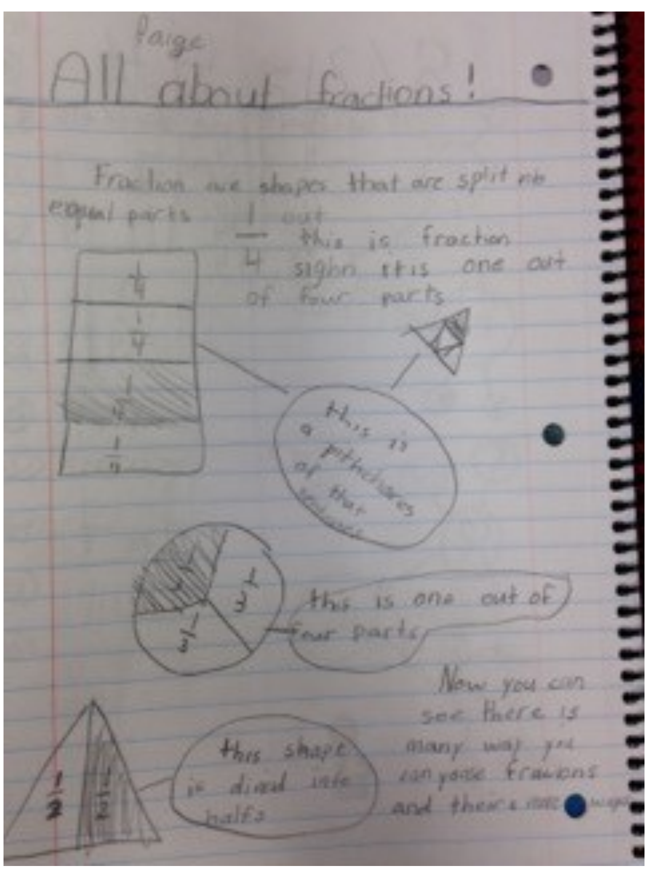
2. I make sure to include a "rectangle divided into ten parts". Most will realize they've made a ten frame… something we work with often. BUT, some will divide it into ten rectangles..also correct! Good discussion! This activity might be repeated on more days.

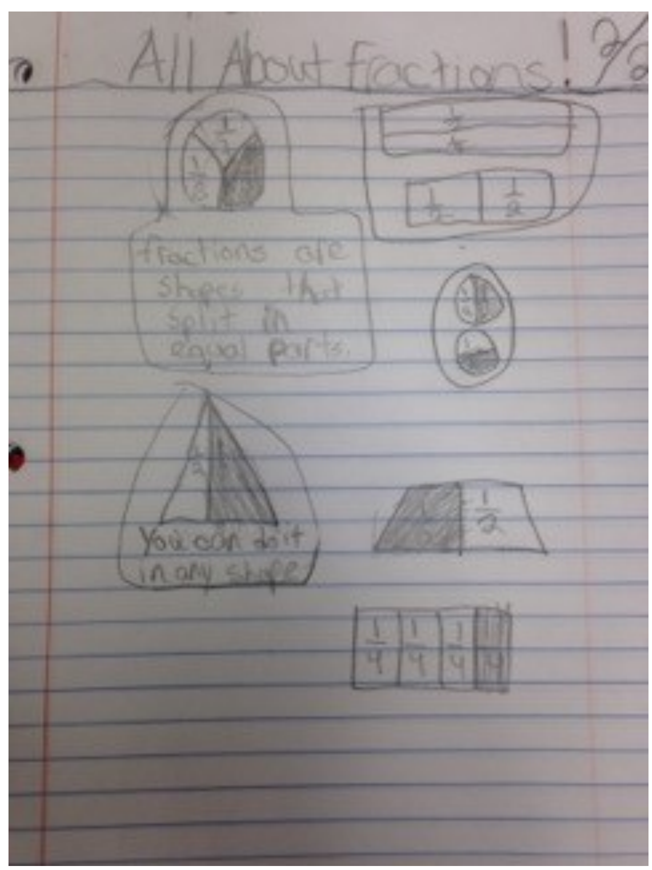


3. Lead students to eventually shade in all of their shape, adjusting the fraction beside it, and recognizing that when all parts are shaded and the same number is on both the top and bottom of the fraction it is a WHOLE.

Assess

We have a common formative assessment for all of second grade, but I also like to have them do a page in their math journal that is "All About Fractions", and just have them write all they can (with pictures, of course!)

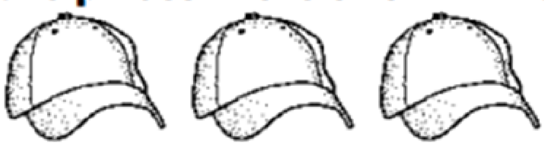




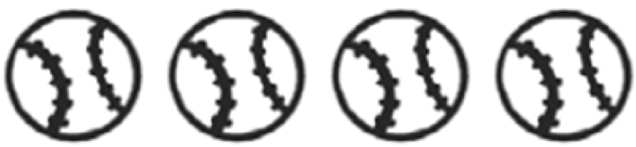
An elementary lesson, grade 2, from insidemathematics.org; Full lesson and additional resources including rubrics and work samples can be found at http://www.insidemathematics.org/assets/common-core-math-tasks/pam's%20shopping%20trip.pdf

Pam's Shopping Trip

Pam's baseball team needs some new equipment. At the store, the prices were shown like this:



$15. **Caps sell 3 for $15.00**



$12. **Balls sell 4 for $12.00**

1. Eight balls cost $_________

Show how you know your answer is correct.

2. One cap costs $ ________

Show how you know your answer is correct.

3. Two caps cost $_________

Show how you know your answer is correct.

Pam has $25. If she only buys caps,

4. What is the greatest number of caps she can buy? ________

Show how you know your answer is correct.

Pam has 4 players on her team who need one new cap and one new ball.

5. How much will that cost all together? $_________

Show how you know your answer is correct.

Using Pattern Tasks to Develop Mathematical Understandings and Set Classroom Norms

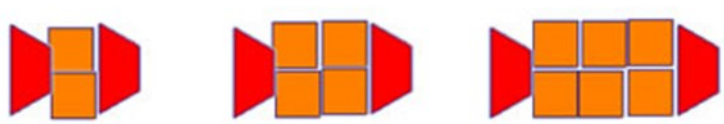
from August 2007 on Mathematics Teaching in the Middle School

The capacity to reason algebraically is critical in shaping students' future opportunities and, as such, is a central theme of K–12 education (NCTM 2000). One component of algebraic reasoning is "the capacity to recognize patterns and organize data to represent situations in which input is related to output by well-defined functional rules" (Driscoll 1999, p. 2). Geometric pattern tasks can be a useful tool for helping students develop algebraic reasoning, because the tasks provide students with opportunities to build patterns with materials such as toothpicks or pattern blocks. These materials help students "focus on the physical changes and how the pattern is being developed" (Friel, Rachlin, and Doyle 2001, p. 10). Such work might help bridge students' earlier mathematical experiences and lay the foundation for more formal work in algebra (English and Warren 1998; Ferrini-Mundy, Lappan, and Phillips 1997; NCTM 2000). Finally, the relationships between the quantities in pattern tasks can be expressed using symbols, tables, and graphs, as well as words. Thus, pattern tasks can also give students opportunities to make connections among representations—a key component in developing an understanding of function (Knuth 2000).

In addition to their potential to help students develop algebraic reasoning, pattern tasks may also be a powerful tool in helping establish classroom norms and practices at the beginning of the school year. In this article, we consider the mathematical and social purposes that pattern tasks can serve by examining one middle school's pattern revolution—a unit on patterns that is used to launch each school year

A Pattern Revolution

During the first few weeks of each school year, students in the seventh and eighth grades at Bellfield Middle School explore a series of geometric pattern tasks. Teachers work together to identify tasks from a variety of sources (e.g., journals, materials from teachers' master's-level course work and professional development experiences, and other curricular materials) and decide which tasks to use at each grade level to ensure that students explore different ones each year. Even with these departmental decisions, teachers have flexibility in determining the amount of time spent on the unit (usually one to two weeks) and in selecting and sequencing the pattern tasks based on the needs of students. Beginning the school year with a unit on patterns has a number of advantages for students, both with respect to developing their capacity to reason algebraically and to participate in a learning community. These advantages are explored in the sections that follow.

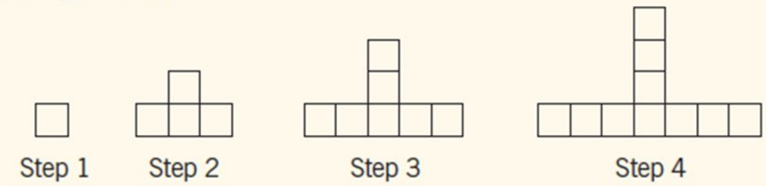


Pattern Tasks Accessible to All

All students, regardless of prior knowledge and experiences, can explore pattern tasks. For example, in solving the Upside Down T Pattern task (shown in fig. 1), some students may build subsequent steps using square tiles or draw the next steps on grid paper, some may make a table and look for numeric patterns, some may view the pattern in one of the ways shown in figure 2, or others may simply notice the recursive "plus 3" pattern. Hence, all students can do something mathematical when presented with a geometric pattern. One teacher noted that regardless of your background, you can have the brightest kid in your class and the one who is struggling feel success from the first two weeks. "So it makes everybody feel like they're on kind of an even playing ground." Once a student has a foothold on solving the task, the teacher is then positioned to ask questions to assess what the student understands about the relationships in the task and to advance students beyond their starting point.

Fig. 1 The Upside Down T Pattern task

Directions: Use the pattern below to answer the following questions. Please show all your work.



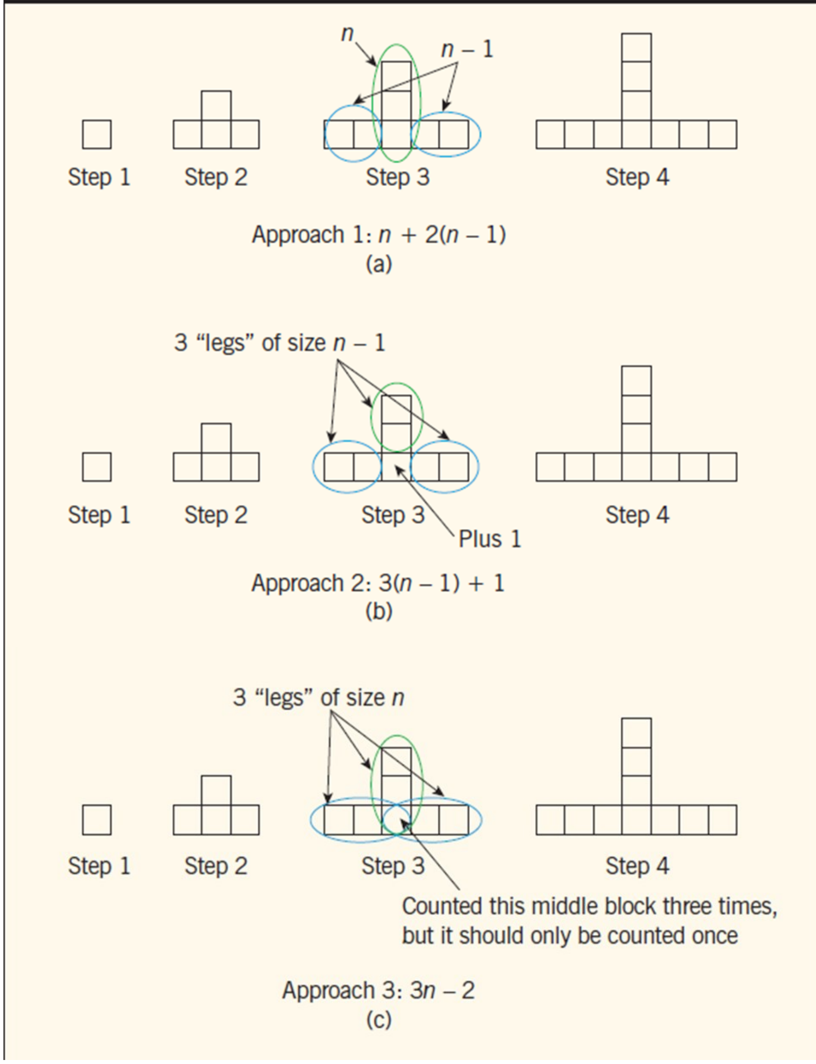
a. Draw the next two steps in the Upside Down T Pattern.
b. How many total tiles (i.e., squares) are in step 5? Step 6?
c. Make some observations about the Upside Down T Pattern that could help you describe larger steps.
d. Sketch and describe two steps in the pattern that are larger than the 10th step.
e. Describe a method for finding the total number of tiles in the 50th step.
f. Write a rule to predict the total number of tiles for any step. Explain how your rule relates to the pattern.
g. Write a different rule to predict the total number of tiles for any step. Explain how your rule relates to the pattern.

In addition, students who have opportunities to explore and discuss relatively simple linear patterns, such as the Upside Down T Pattern, will be able to draw on their work as they encounter more complex items, such as the S Pattern task (shown in fig. 3), which grows in two directions. For example, students who created solutions such as those in figure 2a or 2b (in which the Upside Down T was broken into different chunks) might try a similar strategy for the S Pattern (e.g., breaking the S into a top row of size $(n + 1)$; a bottom row of size $(n + 1)$; and a rectangle, or square, with dimensions $n \times n$). Since strategies are shared publicly, all students have opportunities to make sense of and draw on a range of different strategies in subsequent work.

Tasks Can Be Revisited

Pattern tasks that are explored at the beginning of the school year can become memorable for students and can be referenced as the year progresses. In so doing, these tasks further develop students' understandings of key algebraic ideas. For example, once students have some experience graphing linear equations in two variables, students could draw a graph that shows the number of tiles in each step as a function of the step number in the Upside Down T Pattern task. Students might then consider how the "plus 3" pattern they noticed at the beginning of the year relates to the graph—thus making important connections between the change in successive steps in the pattern, the rate of change in a linear relationship, and the slope of the graph. Alternatively, if students have explored both linear and nonlinear growth patterns (such as the Upside Down T Pattern

Fig. 2 Three visual approaches for determining a generalization for the Upside Down T Pattern task



Shown in fig. 1 and the S Pattern shown in fig. 3), they might be asked to graph both and to compare and contrast the graphs and the expressions. This could lead to a discussion about how the underlying nature of the pattern (i.e., the growth rate) helps predict whether or not the graph is linear.

A Context for Discussing Multiple Solution Strategies

There are usually many different, yet equivalent, ways of expressing the relationship between two variables in a pattern task. For example, in solving the Upside Down T Pattern task, students might determine at least three different symbolic ways of expressing the relationship between the step number and the number of tiles in each step (shown in fig. 2). From a mathematical perspective, this also provides an opportunity to discuss why these expressions are equivalent and how you could justify it. For example, in justifying why the expressions that generalize the relationship in the Upside Down T Pattern

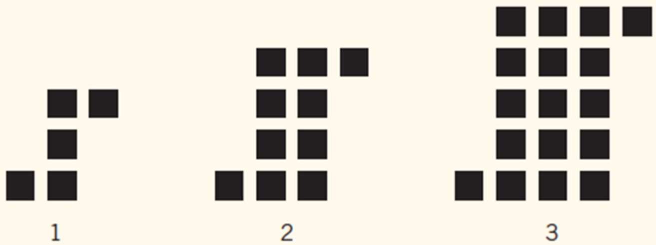
$$n + 2(n - 1),\ 3(n - 1) + 1, \text{ and } 3n - 2$$

are equivalent, students would have opportunities to use algebraic skills, such as combining like terms, using the distributive property, and substituting values for n in each expression and determining whether all the expressions produce the same answer.

In addition, as students explore pattern tasks individually and in small groups, the teacher can challenge them to determine different ways of viewing, describing, and generalizing the pattern. Thus, students learn that the teacher values different ways of thinking about the same problem, and that solving a task in one way is not sufficient. From the teacher's perspective, solving tasks in multiple ways also encourages students to become flexible in their thinking. This skill will benefit students as they encounter problems beyond pattern tasks.

Fig. 3 The S Pattern task

The first three figures in a pattern of tiles are shown below.



Write an equation that could be used to define *any* step in the pattern.

Developing Classroom Norms and Practices

Generating and sharing multiple solutions also provides a way to help students learn how to participate in the classroom community. For example, instead of the teacher beginning the school year by giving students a list of classroom rules, students learn important norms and practices through participation in a community where the practices are reinforced as students engage in mathematical work. For example, beginning on the first day of the pattern unit (usually the first day of school), students are assigned to groups and encouraged to both talk with and listen to one another. Students are consistently asked to look for alternative ways to solve problems and to question the teacher and one another when they do not understand. Students realize that the teacher is a source of support and encouragement but is not someone who is there just to give out answers.

Sharing solutions also serves to establish key aspects of classroom culture. To illustrate how norms and practices such as accountability, clarity, and respect might be established, we zoom in on a classroom in which students are sharing solutions to the Garden Pattern task (shown in **fig. 4**). The students worked on the task in their small groups for about fifteen minutes and then participated in a fifteen-minute whole-class discussion that served two purposes: (1) to determine the extent to which there was consensus on the answers to parts a, b, c, and d; and (2) to make public different ways of determining the number of white squares, given any number of black squares. Beth is the first student to go to the overhead projector and share her thinking about how she determined the number of white squares in each step.

Teacher: OK, Beth, go ahead, tell us how you figured, and everybody pay attention.

Beth: [She walks to the overhead projector, which contains a transparency of the first three steps in the pattern.] You multiply by two and add six.

Teacher: You multiply *what* by two?

Beth: The black squares.

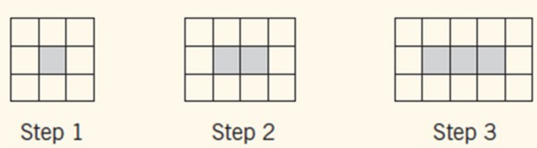
Teacher: Write it down, somewhere. On the top or something. What is she going to multiply by two, Adam?

Adam: [He goes to the overhead projector and writes, "Multiply black squares by 2, add 6."]

Teacher: OK, what did he add there? Does anybody see the little scribble? What did he say?

Fig. 4 The Garden Pattern task

Study the pattern below of forming a row of black squares surrounded by white squares. As you answer the questions, record any patterns you notice.



a. Assume the pattern continues, and draw the next step in the pattern.
b. How many white squares will be in the 5th step?
c. If there are 7 black squares in a row, how many white squares will there be?
d. How many white squares will surround 50 black squares?
e. How many black squares will be in the row if there are 100 white squares surrounding them?
f. How many black squares will be in the row if there are 71 white squares surrounding them?
g. Generalize the number of white and black squares for any step (write a rule, make an equation, state a fact using a step, and so on).

Sara: Black squares.

Teacher: Black squares. You multiply the black squares by two, and then add six. Can you show us on the diagram? Where do you see it on the picture? Where do you see that, to multiply by two? Show on there. Show on the picture. You can write on it [the transparency].

Beth: [She demonstrates her method on step 1, **fig. 5**.] There's one, then one square times two equals two, plus six, equals eight, and then, it's eight squares.

Teacher: OK, you add six. Where is the constant of six?

Beth: Because there's three on each side.

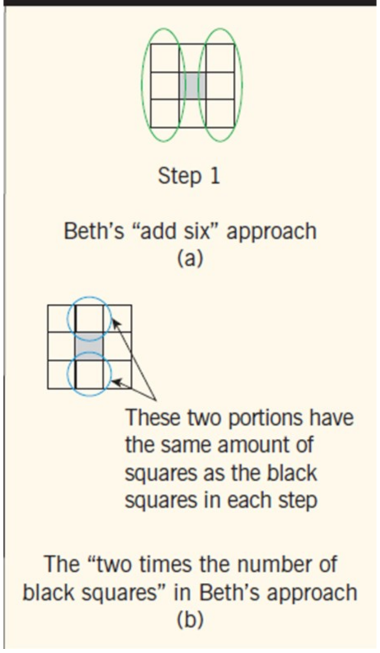
Teacher: Circle them for me.

Beth: [She makes circles around the squares on the sides of step 1, as shown in **fig. 5a**.]

Teacher: One, and the two—where's the two? Two ones, are where?

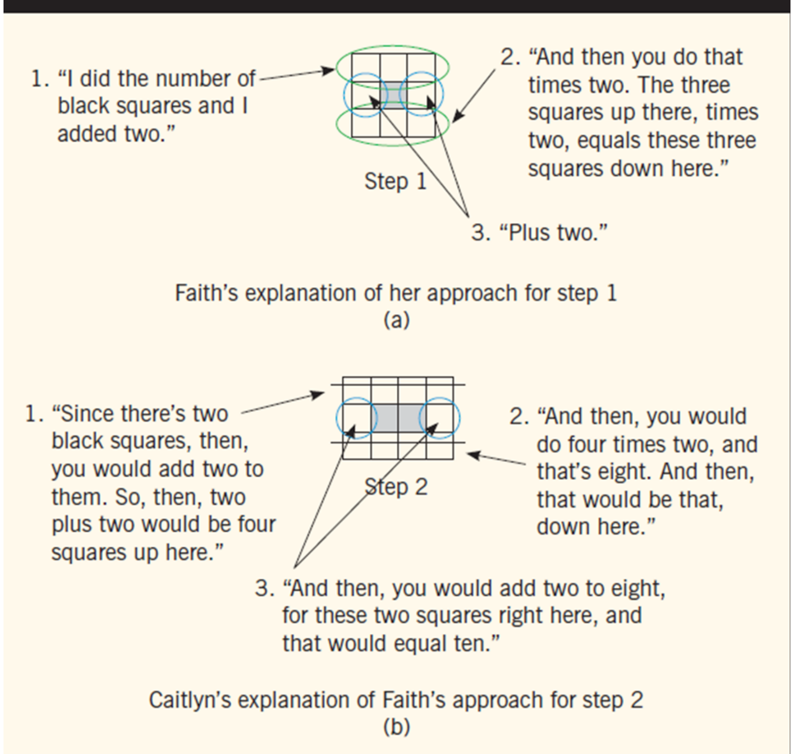
Beth: Right there, and right there [points to the middle square of the three squares on the top row and the bottom row of step 1, as shown in **fig. 5b**.]

Fig. 5 Beth's approach to the Garden Pattern task



After Beth's presentation, the teacher presses students to express Beth's way of viewing the pattern symbolically as $y = 2b + 6$, where b is the number of black squares and y is the number of white squares. The teacher then solicits a second method from the class, and Faith volunteers to share her thinking. Faith describes her approach on step 1 (shown in **fig. 6a**), which makes use of chunking the garden pattern into top and bottom rows that each contain $n + 2$ white squares, plus two white squares (one to the left of the black square and one to the right of the black square). When she is finished with her explanation, the teacher comments, "OK, I don't think they [the students in the class] understood that." Faith describes her approach about how she determined the number of white squares in step 1 again, and the teacher questions Faith and others about her approach.

Fig. 6 Faith's approach to the Garden Pattern task



Teacher: Where's your plus two? Show the class where plus two is.

Faith: These two right here [points to the white squares to the left and right of the black square in step 1], because they're the two remaining squares that you haven't added already.

Teacher: OK, how about the next one? Did anybody pick that up? Can anybody do the next one? All right, so Caitlyn, so show us on the second one. See if you understand what Faith was doing.

Caitlyn: Oh. Oh, yeah.

Teacher: Draw on it. Mess it up. Go ahead.

Caitlyn: [She walks to the overhead projector and demonstrates Faith's method on step 2.] Since there's two black squares, then, you would add two to them. So, then, two plus two would be four squares up here [draws a line across the four white squares that make up the top of step 2, shown in **fig. 6b**].

Teacher: OK.

Caitlyn: And then, you would do four times two, and that's eight. And then, that would be that, down here draws a line across the four white squares that make up the bottom of step 2]. And then, you would add two to eight, for these two squares right here [points to the white squares to the left and right of the two black squares], and that would equal ten.

The teacher then demonstrates Faith's way on the third step of the pattern. At the end of class, Phoebe presents a third rule for finding the number of black and white squares in the Garden Pattern.

Teacher: Did anybody else come up with any other strategies?

Phoebe: OK, well, mine's kind of confusing. It's kind of like Faith's but not. . .

Teacher: OK. Well, say it.

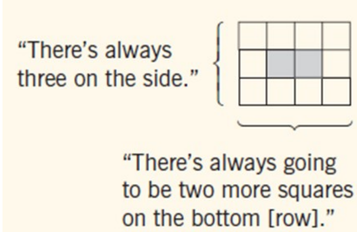
Phoebe: [She goes to the overhead projector and uses step 2 to demonstrate her method, shown in **fig. 7**.] OK, here's always going to be two more squares on the bottom [row].

Teacher: Draw on it.

Phoebe: There's always going to be two more squares down here, than there is right here. So, I knew that—this was for the fifty [black squares] one [question d in **fig. 4**].

Teacher: This is for the fifty [black squares] one [question d in **fig. 4**]. OK.

Fig. 7 Phoebe's approach to the Garden Pattern task



Multiplying the length of the bottom row by three gives you the area. You have to subtract that by the number of black squares.

Phoebe: So, I knew that there was going to be fifty-two on the bottom, for the fifty problem, 'cause there's fifty black squares. And, so I took fifty two times three, these three,'cause there's always three on the side, and I got a hundred fifty-six. And then, since there's fifty, which gives you the area, and since there's fifty colored in, then you have to subtract that by fifty, and that gives you a hundred and six.

Teacher: Oh! That was pretty creative. She took the whole figure, and then subtracted out the area in the middle. Oooh, I like it.

Several aspects of these excerpts are noteworthy. First, students explain and clarify their thinking with the help of comments and questions posed by the teacher. For example, in the beginning of her explanation, Beth tells the class to "multiply by two and add six." The teacher then asks, "You multiply *what* by two?"—a question that helped to clarify Beth's initial statement. In addition, after Faith demonstrates her method on step 1, the teacher presses her for further clarification by commenting, "OK, I don't think they [the students in the class] understood that." With help from the teacher, Faith then provides an explanation that is more clearly connected to the pattern. The teacher also ensures that a clear written record of students' thinking is created by asking students to draw and record their ideas on the transparency. Through these moves, students also learn which aspects of their explanations should be recorded so that everyone in the class can understand the presented strategy.

In addition, students in the class are held accountable for making sense of solutions presented by others. For example, during Beth's presentation, the teacher asks other students to help clarify statements made by Beth and Adam. After Faith demonstrates her method on the first step of the pattern, the teacher holds students accountable by asking if anyone can apply Faith's strategy to find the number of squares in the next step in the pattern. In response to this query, Caitlyn explains Faith's method on the second step of the pattern. Thus, students learn that they are responsible not only for explaining their own thinking but also for carefully listening to, making sense of, and explaining the ideas shared by other students in the class.

During this discussion, the class is also developing important norms regarding respect. The teacher shows respect for her students' mathematical abilities by positioning them as authors of mathematical work. For example, when Faith offers an explanation that the teacher feels may be unclear to other students in the class, the teacher presses Faith to continue her explanation, instead of taking over the explanation for her. Students also show respect for one another's thinking by applying other students' solution strategies.

Finally, important mathematical ideas are publicly made during the discussion. For example, the solution strategies shared by Beth, Faith, and Phoebe are *explicit* forms of pattern generalization (Friel, Rachlin, and Doyle 2001) (rather than *recursive* strategies). That is, the students' generalizations represent the relationship between the step number and the number of white tiles such that the input is related to the output by a well-defined functional rule (Driscoll 1999). For example, Beth notes that "you multiply [the number of black squares] by two and add six." In addition, students are encouraged to make connections between representations—an activity that helps students make sense of relationships such as linear functions (Friel, Rachlin, and Doyle 2001) and develop a more robust understanding of functions (Knuth 2000). In their work on the Garden Pattern task, the teacher presses students to make connections between their generalizations and the geometry of the pattern.

Conclusion

As we have demonstrated in the article, pattern tasks can provide rich opportunities for students to begin to explore relationships between variables and patterns of change. In addition, as one teacher noted, pattern tasks can help set the tone for the school year by helping students "get used to how you run your classes that might be different from other math experiences." It is also important to note that students at Bellfield Middle School appear to recognize the benefits of the pattern unit. For example, one student noted, "Starting with [the pattern unit] made me feel more comfortable in class. I know I can ask questions and not feel stupid. Now I feel encouraged to participate and I enjoy math more." Another student commented, "I was never good at math and I always started out dreading math class. This year I have new hope because I'm allowed to think differently and I'm having success. I look forward to coming to class every day now.'"

So how does one start such a pattern revolution? At Bellfield Middle School, the pattern revolution began with one teacher sharing her students' work on geometric pattern tasks with colleagues. This student work provided an opportunity for teachers to have grounded conversations about what students were able to do mathematically. Other teachers were intrigued by the pattern tasks, and one by one began asking the teacher to share them. As the pattern unit has evolved over time, teachers have had opportunities to draw on a shared experience that has served as the basis for conversations in the teachers' lounge and at department meetings, as well as a platform for further collaboration and discussion.

It is interesting to note that the teachers at Bellfield Middle School have recently adopted a reform-oriented curriculum—due in part, they claim, to their opportunities to witness firsthand what students can do mathematically when posed with high-level tasks during the pattern unit. Despite the adoption of the new curriculum, teachers continue to launch the school year with the pattern unit, since they feel that it serves important mathematical and social purposes.

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About the authors: **Margaret Smith,** pegs@pitt.edu, teaches at the University of Pittsburgh, Pittsburgh, PA 15260, and is interested in supporting and studying the professional development of teachers. **Amy Hillen,** hillen@rmu.edu, is an assistant professor in the department of mathematics at Robert Morris University in Moon Township, PA 15108, where she teaches mathematics methods and content courses for preservice teachers. **Christy Catania,** christy_catania@fcasd.edu, teaches middle school in the Fox Chapel Area School District in Pittsburgh, PA 15238. She is interested in using patterning tasks in the classroom and working with preservice teachers.

Standard for Mathematical Practice #4
Model with mathematics

A Middle School Example – Liz Peyser

Through this practice, mathematically proficient students create models to represent the relationships among the quantities in a situation. From the models, they gain insight into how to solve the problem, or what pattern they notice so they can create a new model. This is an exciting Practice that really makes the mathematics "visual" for students by using physical models, tables, graphs, flowcharts, equations and formulas. Students can ask themselves these questions:
"Can I make my life easier by using math to optimize solutions?"
"Is there a way I can demonstrate real world problems using math?"
"Could a mathematical model make the problem situation and potential solution clearer?
(courtesy of Weber State University)

There is a lot of confusion between SMP 4 and SMP 5 – "use tools strategically". Tools themselves can be used to model the math, and then more abstract models can be developed through questioning by the teacher.

Let's consider a problem from a recent NCSM presentation (Illustrating the Standards for Mathematical Practice: Modeling with Mathematics):

"Lunches in our school cost $2 each. How much do 2 lunches cost? 3 lunches? 4 lunches?... 10 lunches? More lunches? Create at least 2 models of this situation. You can choose a physical model, a table, a graph and/or an equation. Your model should show the number of lunches and the cost of the lunches. You should be able to use your model to find the cost of a certain number of lunches."

To "model with mathematics" students are asked to do the following:

To express a situation using mathematical representations such as physical objects, diagrams, graphs, table, number lines or symbols
Operate within the mathematical context to solve the problem
Use the solution to answer the original question; interpret the result in the context of the situation
Improve the model if needed

In this particular example, tools could be used as a physical model, such as chips or markers to present dollars for each lunch. We could have two markers for each lunch. To find the cost of three lunches, we would have 3 groups of 2 markers.

Cost of one lunch

In a different model, a student could create a table of values:

Number of lunches	Total cost
1 lunch	$2
2 lunches	$4
3 lunches	$6
4 lunches	$8
10 lunches	$20
Any lunches	??

In each of these models, the teacher can focus the students on the relationships of the quantities and the pattern that develops, and what aspects the students can glean from each of the models. The teacher can also focus the students to analyze how the models are related to one another:

What aspects do we see from the table?
Where is that shown in the physical model?

Through careful questioning, the teacher can help the students take the pattern and make an abstract, generalized model:

What are they seeing every time a lunch is added?
Is there a way to predict, using their models, what the cost will be of 15 lunches, or 9 lunches?
What is happening every time we have an order of lunches?
Can we create a new model?

Yes, we can create an equation to represent the relationship between the cost of each lunch and the total cost: cost = $2 multiplied by the number of lunches.

Although we are specifically looking at SMP #4, it is easy to see how SMP #2 and SMP #5 are inadvertently included. By creating math models, the students are beginning to move from concrete to abstract and "reason quantitatively" (MP 2). They are also using a variety of tools, and strategically choosing the ones that will help them express the situation (MP 5).

A Middle Grades (6th) Lesson from Mathematics Assessment Project; full lesson with additional resources can be found at http://map.mathshell.org.uk/materials/lessons.php?taskid=588&subpage=problem

Car Skid Marks

Dek and Mani are traffic accident investigators.
Their job is to find out how and why accidents happen.
To do this they gather evidence from the scenes of accidents.

When a car suddenly brakes to a stop, it can leave skid marks.
These marks can be used to figure out the speed of the car.
This might give evidence that the driver was going over the speed limit.

On a dry test track, a car is driven at different speeds, in miles per hour (mph).
Each time it brakes as hard as possible.
The skid length is then measured in feet.
Here are the results:

Speed (mph)	0	19	27	29	37	39	49	54	56	60	66	69	74	76	80	85	89	93	98
Length (feet)	0	20	37	42	61	68	100	120	131	150	180	200	230	240	270	300	330	360	400

The relationship shown in the table looks complicated, so Dek and Mani both try to work out a 'rule of thumb' for estimating the speed of a car from the length of the skid marks:

Dek

I've got an easy rule.

Halve the length of the skid mark in feet.

This gives an estimate for the speed in miles per hour.

Mani

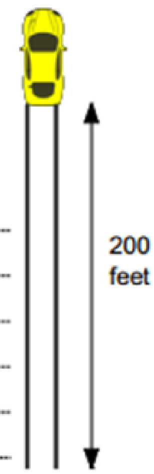
My rule is more complicated.
I use the formula:

$$y = \frac{x}{4} + 30$$

y is the speed of the car in miles per hour
x is the length of the skid mark in feet

Car Skid Marks (continued)

A car was travelling on a dry flat road with brakes in good condition.
The skid marks for the car measured 200 feet.



1. Which rule gives the best estimate for the speed of the car: Dek's or Mani's?
   Show all your work.

2. Dek and Mani argue about which rule is the best one to use for ***any*** traffic accident.
   What is your advice?
   Show your work and explain your reasoning.

I will carefully select a range of skid lengths to test Dek and Mani's rules of thumb.

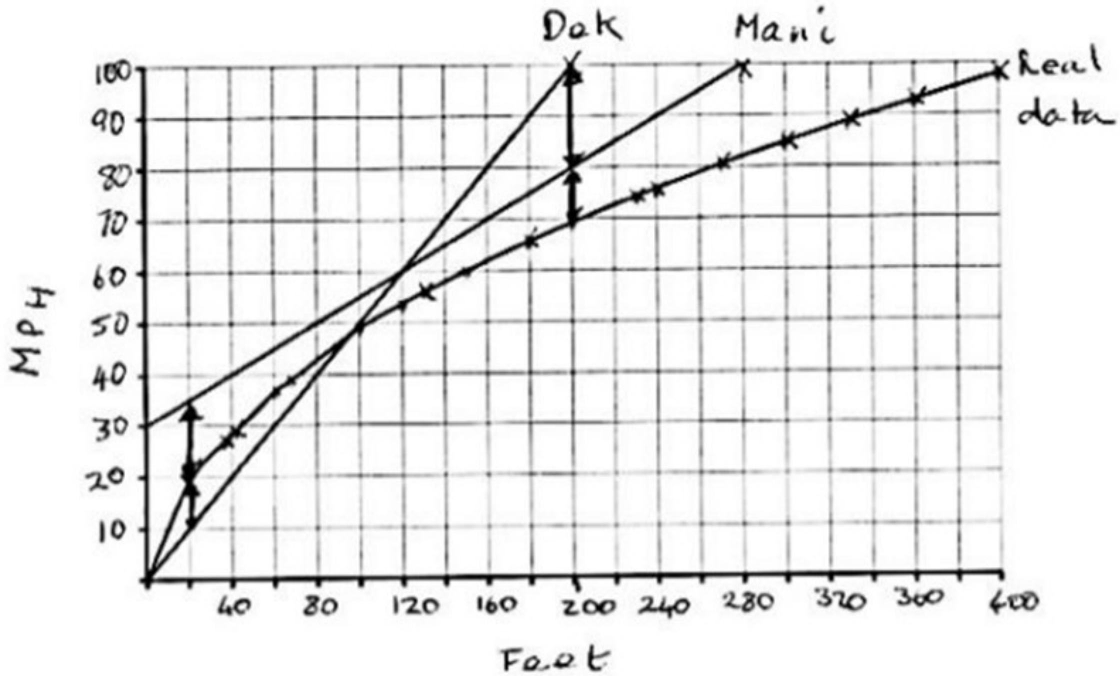
Length of skid	Correct speed	Dek's speed	Mani's speed	Best rule of thumb
0	0	0 (Error 0)	30 (30)	Dek
42	29	21 (Error 29-21=8)	40.5 (40.5-29=11.5)	
100	49	50 (Error 50-49=1)	55 (55-49=6)	
150	60	75 (75-60=15)	67.5 (67.5-60=7.5)	
200	69	100 (100-69=31)	80 (80-69=11)	
240	76	120 (120-76=44)	90 (90-76=14)	
300	85	150 (150-85=65)	105 (105-85=20)	
360	93	180 (180-93=87)	120 (120-93=27)	
400	98	200 (200-98=102)	130 (130-98=32)	

Clearly explain Ezra's method. (You do not need to check Ezra's arithmetic; it is correct)

Use Ezra's method to complete the final column. What conclusion could Ezra make?

How could Ezra improve his work? Fully explain your answer.

Sample Student Work: Leanne



Clearly explain Leanne's method.

What conclusion could Leanne make?

How could Leanne improve her work? Fully explain your answer.

Using Covariation Reasoning to Support Mathematical Modeling

from March 2014 in Mathematics Teacher

For many students, making connections between mathematical ideas and the real world is one of the most intriguing and rewarding aspects of the study of mathematics. In the Common Core State Standards for Mathematics (CCSSI 2010), mathematical modeling is highlighted as a mathematical practice standard for all grades. To engage in mathematical modeling, beginning algebra students must learn to use their understanding of arithmetic operations to make mathematical sense of problem situations and to relate this sense making to functions represented by equations, tables, and graphs. The word problems commonly used in beginning algebra courses give opportunities to practice mathematical modeling. Further, the ability to reason with quantities as well as numbers is an important capacity for students to develop.

Quantities are conceptions of things that can be measured, such as distance or time. The measure of a quantity has a defined unit and a process for assigning a number that represents the proportional relationship between a particular value of the quantity and the unit (Thompson 2011). Quantitative reasoning is different from numerical reasoning because quantitative reasoning involves a clear mental image of how quantities are related (Thompson 2011). Someone who quantitatively understands the average speed of a sprinter who runs 100 meters in 4 seconds might imagine the 100-meter track divided into 4 sections, each 25 meters long, and then imagine the runner traversing one section during each second. By contrast, if students compute 100 m/4 s = 25 m/s, they may understand only the arithmetic relationship between the numbers 100, 4, and 25 without understanding why dividing 100 by 4 makes sense in this situation. Quantitative reasoning is a key resource for students who are learning to use algebra to model relationships between quantities that vary.

Two kinds of quantitative reasoning have a special relevance for beginning algebra students. The *correspondence* perspective deals with the question, How is one quantity related to another? A correspondence understanding of speed might be expressed as the rule that relates each value for time with a unique value for distance, such as the equation $y = 25x$, where x represents time and y represents distance. By contrast, the key question for *covariation* reasoning is, How does one quantity change as another quantity changes? A covariation understanding of speed would focus on how distance and time change together—that is, the distance covered increases by 25 meters as the elapsed time increases by 1 second.

Both kinds of reasoning are important goals for algebra students. Correspondence is a fundamental piece of mature reasoning about functions, and covariation is critical for developing the rate-of-change concept. Research shows that covariation is a common entry point into algebra for students (e.g., Confrey and Smith 1995); but traditional approaches to teaching algebra emphasize correspondence and often have little or no treatment of covariation (Smith 2003). Thus, this article focuses on students' use of covariation and how to support it the classroom.

Presented here are two sessions from Ms. Holmes's classroom (the teacher's name is a pseudonym) in which seventh graders intuitively used covariation to begin to make sense of word problems. These passages show how students' covariation reasoning might surface in the classroom and illustrate some of the teaching strategies that Ms. Holmes used to support her students' reasoning. The sessions also provide a foundation for the discussion of classroom strategies, which summarizes research-based strategies for supporting students' use of covariation reasoning to build robust mathematical models.

CLASSROOM SESSION 1

In session 1, a small group of students was working on problem 1 during a lesson spent reviewing for a state achievement test.

Problem 1: Sally needs 3 2/7 yards of fringe to trim each drape. If she has 8 drapes, how much fringe does she need? What operation is used to solve this problem?

(a) addition

(b) subtraction

(c) multiplication

(d) division

The first student started by guessing that the operation was subtraction but after some time changed his mind.

Student 1: I don't think you subtract now.

Ms. Holmes: What do you [*other*] guys think?

Student 2: Um . . . divide?

Student 3: Yeah.

Ms. Holmes: Divide? Why?

Student 2: Because . . . [*thirty-five seconds elapse*] . . . I don't know.

Rather than challenging student 1, Ms. Holmes asked for other students' ideas. She also pressed students to justify their answer of division.

To help the students make progress, Ms. Holmes next read the problem out loud, asked the students to explain the problem in their own words, and had them draw a picture of the situation. After a few minutes, the students were still stuck, so she asked a sequence of questions:

Ms. Holmes: So, if Sally had 1 drape, how many yards would she need?

Student 1: Three and two-sevenths [3 2/7] . . .

Ms. Holmes: All right. What if she had 2?

Student 1: She would need . . . twice that 3 2/7.

Ms. Holmes: [*nods*] What if she had 3?

Student 1: Twice . . . I mean, 3 times that.

Ms. Holmes: So, what are you doing each time?

Student 1: Multiplying . . . Oh! . . . So you multiply. It's multiplication.

Ms. Holmes then asked the two other students to explain why multiplication was the appropriate operation.

Discussion of Session 1

Each of Ms. Holmes's questions asked student 1 to compare a quantity of drapes with the corresponding quantity of fringe, so each of these questions is about correspondence. Neither Ms. Holmes nor the students used a table of values for this problem, but it is natural to imagine a hypothetical table (see **fig. 1**). In such a table, each of Ms. Holmes's questions asks about values within a single row.

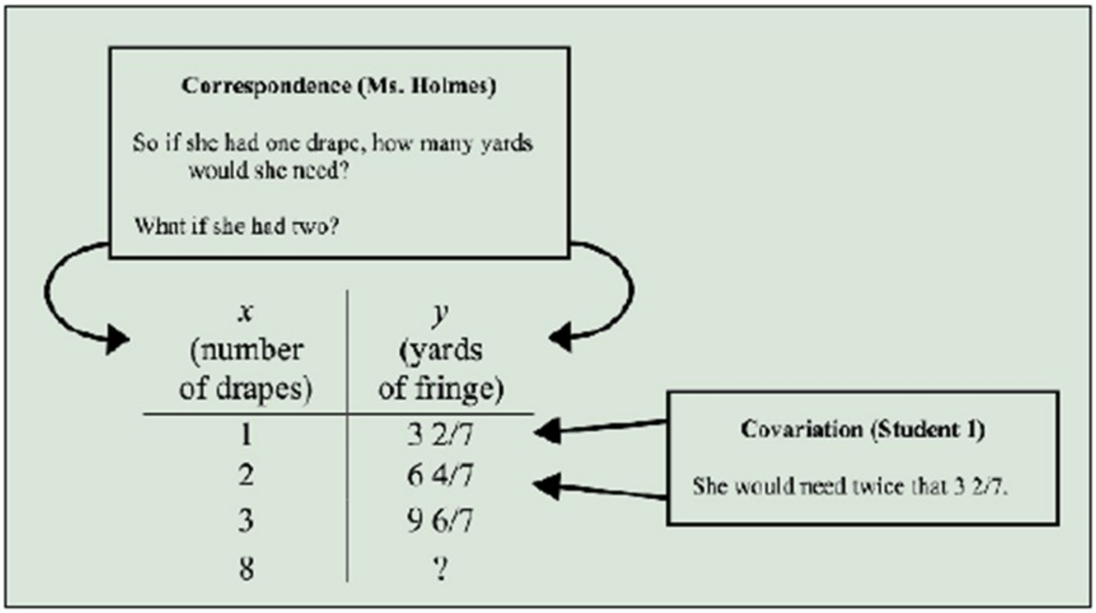


Fig. 1 The teacher intends to build a correspondence between quantities in the same row. The student sees a covariation between rows.

What is interesting about this episode is that the sequence of questions implied a comparison between the paired quantities. The implicit reasoning supporting student 1's statement that 2 drapes would need "twice that 3 2/7s" is that 2 drapes are twice as many as 1 drape. The student's responses indicate that he recognized that as the quantity of drapes doubles and triples, the quantity of fringe is multiplied by 2 and then by 3. This reasoning can be understood as a comparison between rows in the hypothetical table of values (see **fig. 1**).

The final part of the episode involved looking back over the sequence of examples. Ms. Holmes's final question was probably intended as a general correspondence question, with "each time" referring to each correspondence between the number of drapes and the amount of fringe. The student likely interpreted "each time" to refer to each new pair of drapes and fringe and compared the new pair with the initial one. In any event, by asking a sequence of specific questions and then asking the student to reflect across these examples, Ms. Holmes was able to build this student's covariation reasoning and help him establish a meaningful mathematical model of the quantities in problem 1.

CLASSROOM SESSION 2

The class discussion of problem 2 occurred on the same day but during a different period and with different students. Earlier in the year, these students had worked on distinguishing directly and inversely proportional relationships and on writing linear equations using tables of corresponding *x*- and *y*-values.

Problem 2: Juan can clean up after the party in 2 hours if he works alone, but he hopes his friends will help. Write an equation relating the number of people (*x*) and the amount of time (*y*) it would take to clean up if everyone works at the same rate.

One student initially guessed that the equation was y = 2x. However, Ms. Holmes pointed out that this equation would not work because as the number of people increased, the amount of time should decrease. The students were still stuck, so to direct the students' reasoning about the problem, Ms. Holmes constructed a table on the whiteboard that included three values for x (the number of people): 1, 2, and 4 (see fig. 2). As she questioned the students, they readily agreed that one person would take 2 hours and that 2 people would take just 1 hour, but they disagreed about the time it would take 4 people.

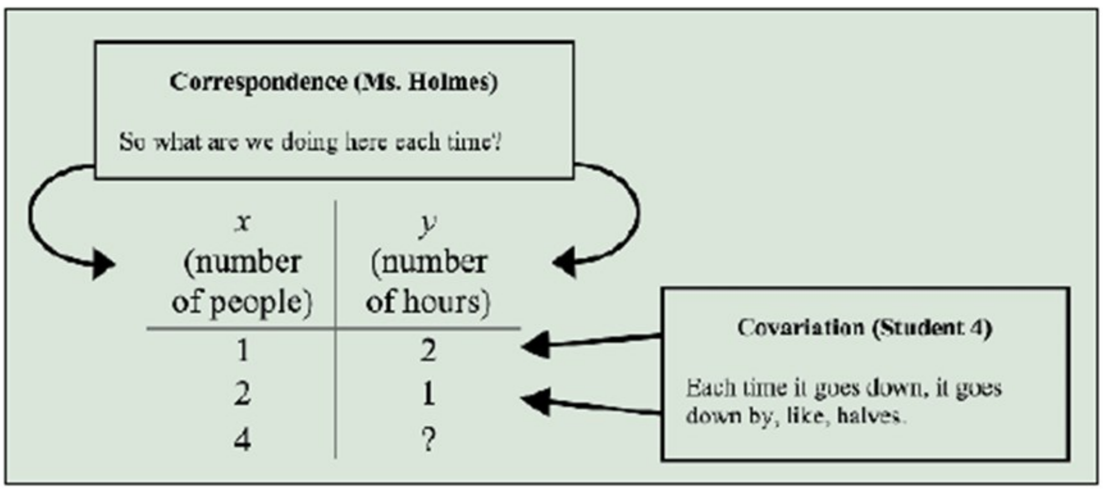


Fig. 2 The student generalizes (incorrectly) using covariation.

Ms. Holmes: What if you had 4 people?

Student 1: Thirty [30] minutes.

Ms. Holmes: How are you getting that?

Student 1: It's . . . um . . .

Student 2: It'd be 15 minutes . . . because it's half of . . . because if you have 3 people, it would be 30, and 4 it would be 15.

Ms. Holmes: Explain to me what you're thinking.

Student 3: x divided by two equals y.

Ms. Holmes: x divided by 2? [points to the first row of the table in fig. 2] Well, 1 divided by 2 is 1/2 . . .

Student 3: Oh, it's y divided by 2 equals x!

Ms. Holmes: Well, 2 divided by 1 is 2, but 2 divided by 2 . . .

Student 4: [interrupting] Each time it goes down, it goes down by, like, halves.

Ms. Holmes: So, what are we doing here each time [pointing to x and then y in the table]?

Student 5: Going down by a half each time.

Only a few minutes were left in the period, so Ms. Holmes went on to remind the class of the equation $y = k/x$ (the correspondence rule). She showed that this rule was algebraically equivalent to $yx = k$. Finally, she demonstrated that the product of x and y was always 2 in this problem.

Discussion of Session 2

In this episode, Ms. Holmes pressed students to explain their thinking and offered counterexamples when students guessed incorrectly. In this way, she enabled students to describe their thinking and also promoted mathematical reasoning.

Just as in classroom session 1, the key was Ms. Holmes's sequence of questions about correspondence. As before, several students were reasoning about covariation in the context of the problem. Student 2 hypothesized that the value for 3 people was 1/2 hour, or 30 minutes, and his hypothesized time for 4 people was half the previous value—1/4 hour, or 15 minutes—perhaps because he believed that each new person would cut the time in half. Student 4 apparently picked up on this idea and made a more general covariance statement.

The final part of the episode involved looking back over the sequence of examples. Ms. Holmes's final question was intended as a general correspondence question, just like her final question of the first episode. Her gestures indicated that "each time" referred to each row relating a specific number of people and the corresponding amount of time. However, student 5 reasoned about covariance instead of correspondence and interpreted "each time" to mean each new row. He compared each new row with the previous one, claiming that time was "going down by a half each time." Unfortunately, the period ended before these students had an opportunity to differentiate correspondence and covariation reasoning about the problem situation or find the correct mathematical model.

STRATEGIES FOR THE CLASSROOM

As we review the classroom episodes and the research literature (e.g., Carlson and Oehrtman 2005; Ellis 2011), several strategies emerge that teachers can use to support students' covariation reasoning:

1. Use a sequence of specific pairs of values to support students' reasoning about the problem.

2. Ask students to describe and explain their thinking about a single pair of values and to compare different pairs of values. Listen carefully for covariation or correspondence reasoning.

3. If students make incorrect claims, ask for other students' ideas. Model quantitative reasoning by providing a mathematical reason or counterexample based in the problem situation that explains why the claim is incorrect.

4. Support covariation reasoning by asking students the following kinds of questions about problem situations (adapted from Carlson and Oehrtman 2005):

- What quantities are changing together, and how are they changing?
- As one quantity increases, does the other quantity increase or decrease?

- As one quantity increases in constant increments, by what amount does the other quantity change?
- As one quantity increases in constant increments, what is the rate of change in the other quantity?

Selecting specific examples (strategy 1) so that one quantity is changing in constant increments might emphasize covariation relationships for students, such as Ms. Holmes's choice of 1, 2, and 3 for problem 1. On the other hand, the choice of a doubling sequence 1, 2, and 4 for problem 2 may have contributed to the students' misunderstanding of the relationship between people and time.

As teachers listen carefully and help students communicate clearly about covariation, there will be opportunities for mathematical exploration that can enrich students' conceptions of functions. For example, students 2, 4, and 5 in episode 2 were actually describing a different function that could be represented with the correspondence rule, $y = 2(1/2)x-1$, or in terms of covariation (y decreases by a factor of 1/2 as x increases by 1). A teacher might ask students in the class to compare this function with the correct function for problem 2: $y = 2/x$.

REASONING ABOUT COVARIATION IS CRUCIAL

Reasoning about covariation is not a crutch; it is a crucial skill. Developing the ability to clearly and explicitly reason about quantities that are changing together will support students in beginning algebra and lay the foundation for later success in mathematics. Students who intuitively use covariation to think about how quantities change may not have access to simple, unambiguous language to describe their thinking. When using tables to develop students' understanding of functions, teachers can help students describe, explain, compare, and relate covariation reasoning between rows and correspondence reasoning within rows.

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About the author: **ERIK JACOBSON,** ejacobsn@uga.edu, is a doctoral candidate at the University of Georgia in athens. Before graduate school, he taught high school algebra in Vermont.

High School/College Lesson

Submitted by Lanee Young
Fort Hays State University

There are times as math educators we do not take or have the opportunity to apply the concepts from our mathematics classes to the real world in a meaningful way. In 2010 Dan Meyer illustrated that word problems from a traditional mathematics text give the students exactly the information they will need, follow a specific example, and can be solved without any real understanding of the mathematics content (Meyer, May 2010). These types of problems from textbooks do not promote the practice standards including modeling with mathematics or provide a valid assessment of the students understanding of the concept. In order to help our students use mathematics in the real world, we must provide experiences for students to practice modeling while interpreting and reflecting upon the results to determine the implications of these concepts in the real world.

Concepts within statistics and probability offer many opportunities for our students to model using mathematics. With the increase of mobile technology, data is more available and potentially more unreliable than ever before. Through the analysis of actual data found on the internet or collected by students, mathematics educators can help to build a group of informed consumers who understand the variability of data and the processes required to make solid conclusions. Providing students the opportunity to determine if an outcome happens by random chance or because of some external force is an important life-long skill for students to practice. The following activity (adapted from https://stat.duke.edu/~gp42/sta101/proj.html) has been developed for students to practice collecting, interpreting, and analyzing data to make inferences and justify conclusions over a topic of their choosing.

PURPOSE

This unit is intended to help the instructor assess how well students are able to:

- Design an experiment,
- Analyze data
- Create and use regression equations
- Perform a hypothesis test
- Create and interpret confidence intervals.
- Use technology
- Write and present results and conclusions to others.

TIME NEEDED

15 minutes in class for three class periods
One Class period for poster session
Outside of class time varies

PROJECT OUTLINE

1. Students determine an idea that will be interesting for their group to study and write a hypothesis concerning the expected results. This information will be submitted to their instructor before proceeding to step 2.
2. Determine the best way to collect the data: experiment, observational study, or survey

   Write your project proposal. Be specific when describing what question(s) you want to investigate, how you plan to get data, and how you might be able to analyze it. The instructor will return the proposals to you with comments. Be sure to address the following questions:

   *What is the topic of your project?
   *Why did you choose this topic?
   *What are the main issues or problems you plan to address?
   *Research background information concerning your topic. Look for similar studies, results, and information you can use as data for the null hypothesis. List the websites where you found valid information.

*Describe the data and how it will be collected (experimental design, observational method, or survey). (Include if the data is quantitative or qualitative and continuous or discrete. Be sure to include if you data is going to be nominal, ordinal, ratio, or interval.)

*What questions do you have concerning this project?

4. After your project proposal has been approved by your instructor you may begin collecting data.
5. Throughout the course we have learned various methods to display and analyze data. You must include a hypothesis test and confidence interval in your data analysis. Additionally you must choose one other topic from the semester to use as part of your analysis. Please be aware that simply stating a measure of center or drawing a graph is not an analysis of data. Be thorough in your analysis.
6. Create a poster for our poster session including the following elements:

*__Statement of the problem__: Describe the questions you addressed, why it is important to the population, and why you chose it.

*__Data collection__: Explain how you collected data including the type of study, questions asked, method, and response rates.

*__Analyses:__ Describe the analysis you did. Justify your choices

*__Results__: Present relevant descriptive statistics (e.g., number of men and women surveyed, if that is important). You may include tables or graphs.

*__Conclusions__: Answer your question of interest. Describe the results you found from hypothesis test, confidence interval, and additional methods.

*__Discussion__: What implications do your results have for the population from which you sampled? What could be done to improve the study if it was done again? How might you extend the study for future research??

You should strive to make the poster clear and appealing. Avoid unnecessary clutter. You want your audience to understand the topic and the conclusions you have made in a quick concise manner.

Although the studies the students performed were not perfect there was evidence that learning had taken place throughout the semester. One group in particular studied how much students spent at the coffee shop on campus. Although the data suggested that the average amount spent at the coffee shop was more than four dollars, students recognized potential bias might result from students trying to burn "dining dollars" before the semester ended. This showed some ability to analyze the data and determine if their results made sense.

On the day of the poster session students were required to observe all student posters to determine what other groups did well, what could be improved, and finally something that was learned from each groups study. The students' evaluation of the other posters was also used as a valuable assessment for the instructor. While critiquing each other's projects some students were excited about pretty decorations others recognized flaws in reasoning and offered suggestions for changes to the current study or questions for future studies.

Since this was the first time the instructor had tried an activity like this in a statistics course, students were also asked to evaluate what they learned from the project and if they would recommend it for other classes. Although the instructor was expecting that students did not enjoy or would not recommend the project to others, students were overall positive stating "we liked it was something different than homework and tests, enjoyed choosing their own topic, and learned how to apply what we learned in class to a real life situation."

Throughout this project students were able to apply the mathematics to solve everyday problems. By analyzing their data, students were able to draw conclusions and interpret their results which was mutually beneficial for the instructor and students. Through the incorporation of modeling, students were given a better understanding of the mathematical content that was studied throughout the semester.

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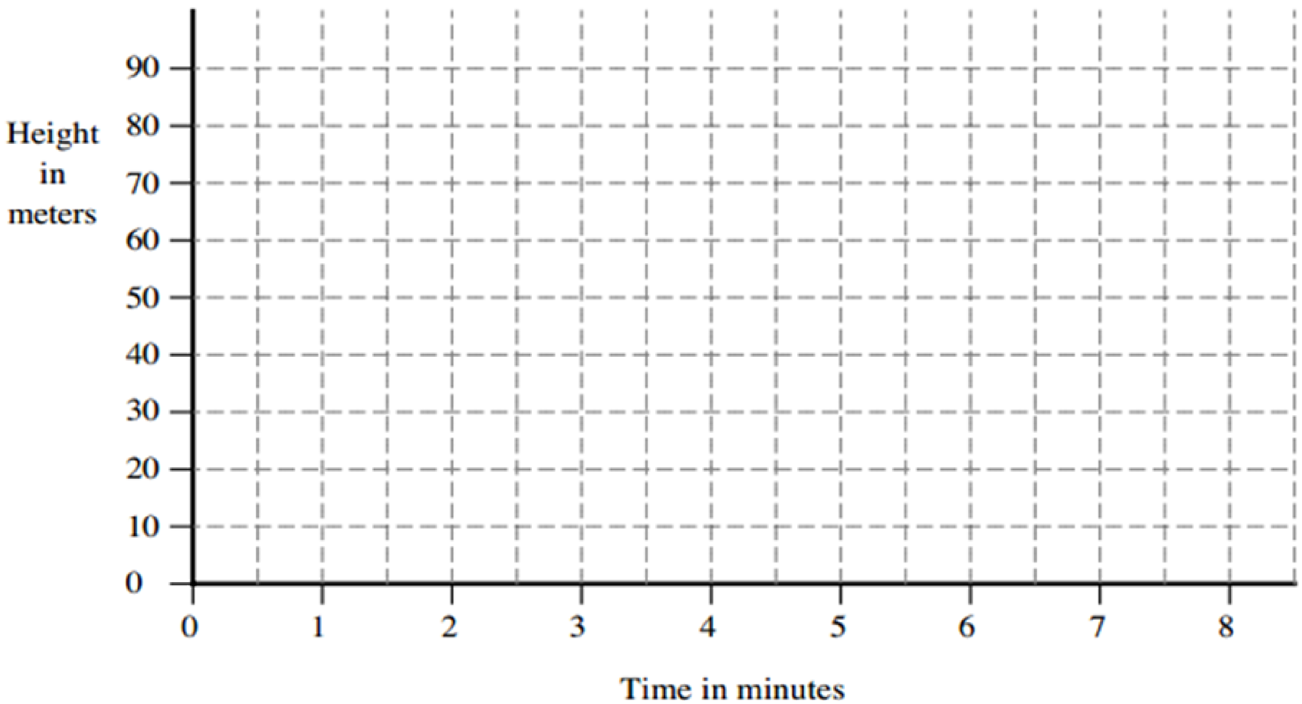
Ferris Wheel

A Ferris Wheel is 60 meters in diameter and rotates once every four minutes.

The center axle of the Ferris Wheel is 40 meters from the ground.

1. Using the axes below, sketch a graph to show how the height of a passenger will vary with time.
   Assume that the wheel starts rotating when the passenger is at the bottom.



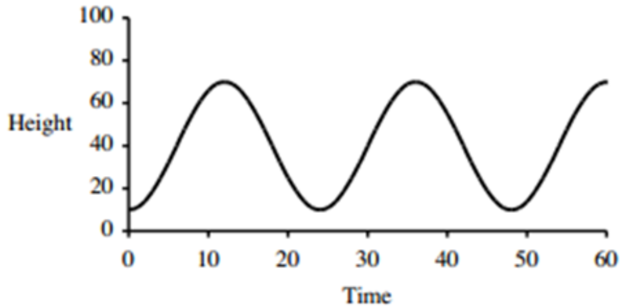
2. A mathematical model for this motion is given by the formula:

   $h = a + b \cos ct$ where h = the height of the car in meters
   t = the time that has elapsed in minutes
   a, b, c are constants.

   Find values for a, b and c that will model this situation.

Card Set A: Graphs

A.



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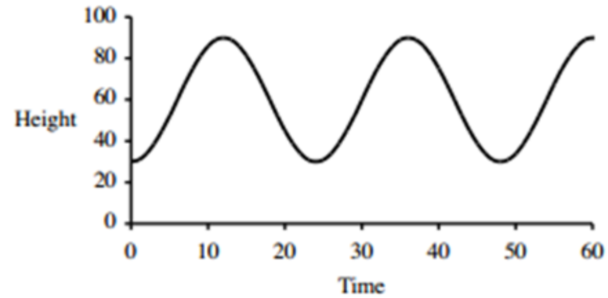

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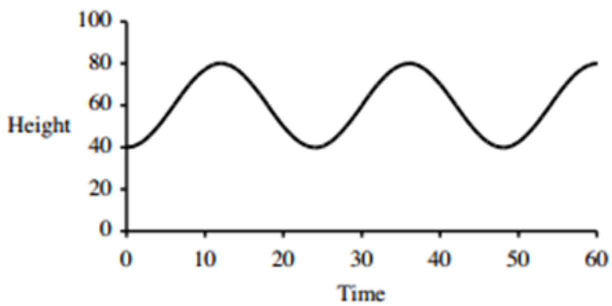

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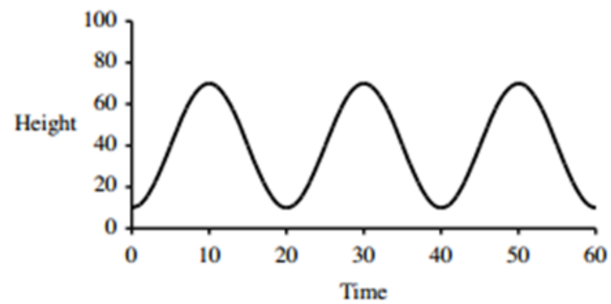

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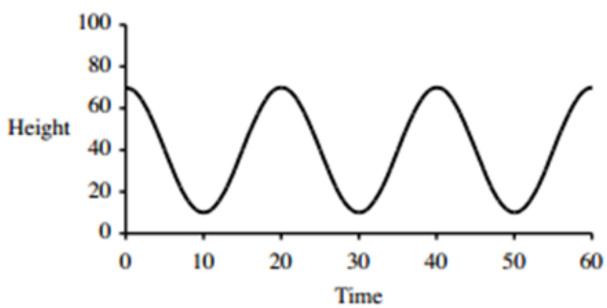

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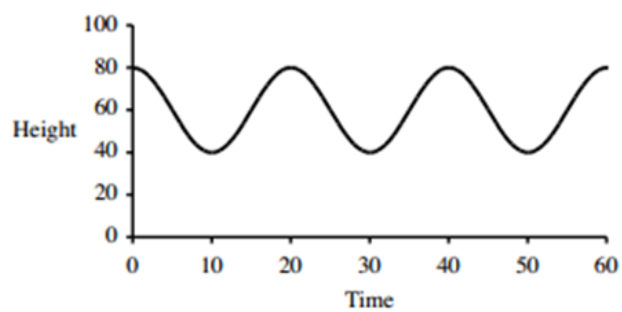

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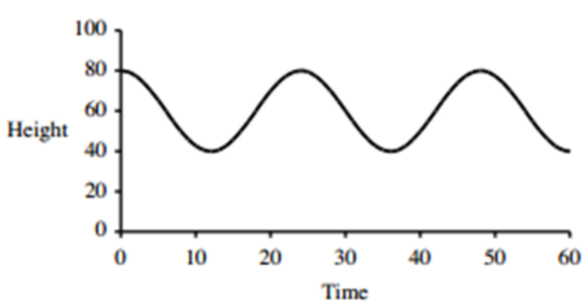

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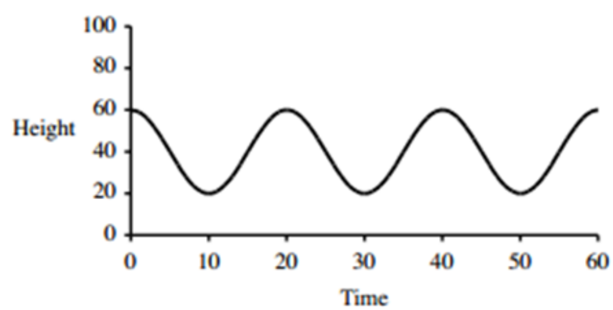


Card Set B: Functions

1. $h = 40 + 30 \cos 18t$	2. $h = 60 + 20 \cos 15t$
3. $h = 40 - 30 \cos 18t$	4. $h = 40 - 30 \cos 15t$
5. $h = 60 + 20 \cos 18t$	6. $h = 60 - 20 \cos 15t$
7. $h = 40 + 20 \cos 18t$	8. $h = 60 + 20 \cos (15t + 180°)$
9. $h = 40 + 30 \cos (18t + 180°)$	10.

Card Set C: Descriptions of wheels

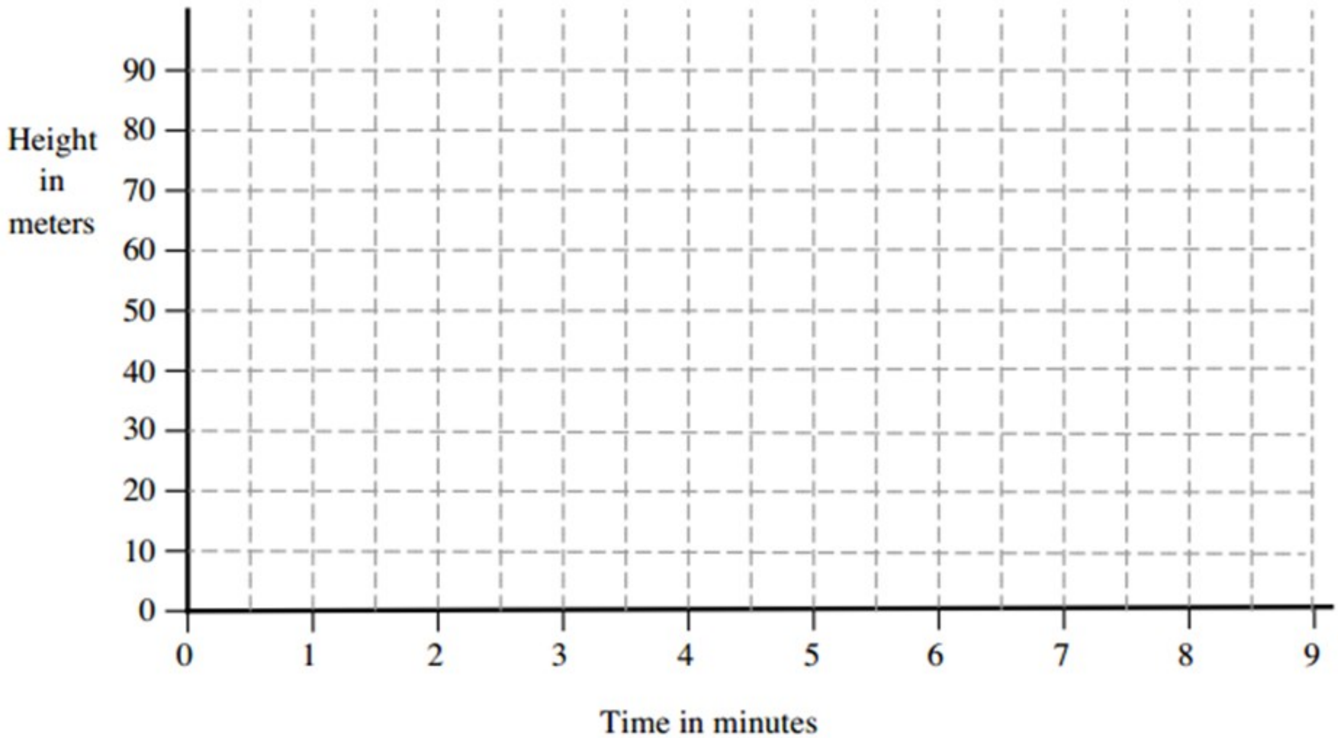
1. Diameter of wheel = 40 m Height of axle above ground = 60 m Number of turns per minute = 2.5	2. Diameter of wheel = 60 m Height of axle above ground = 60 m Number of turns per minute = 2.5
3. Diameter of wheel = 60 m Height of axle above ground = 40 m Number of turns per minute = 2.5	4. Diameter of wheel = 60 m Height of axle above ground = 40 m Number of turns per minute = 3
5 Diameter of wheel = 40 m Height of axle above ground = 40 m Number of turns per minute = 3	6. Diameter of wheel = 40 m Height of axle above ground = 60 m Number of turns per minute = 3

Ferris Wheel (revisited)

A Ferris Wheel is 50 meters in diameter and rotates once every three minutes.

The center axle of the Ferris Wheel is 30 meters from the ground.

1. Using the axes below, sketch a graph to show how the height of a passenger will vary with time.
   Assume that the wheel starts rotating when the passenger is at the bottom.



2. A mathematical model for this motion is given by the formula:

   $h = a + b \cos ct$ where h = the height of the car in meters
   t = the time that has elapsed in minutes
   a, b, c are constants.

   Find values for a, b and c that will model this situation.

Systems of Equations Introduction

Activity provided by Jenny Wilcox

Find the cost of a soccer ball and a jersey. Be prepared to explain your method, and one other method presented in class.

$32

$86

MY METHOD	ANOTHER METHOD

I have used this activity to introduce systems of equations. Usually, almost my entire class uses "guess and check" to approach the task. However, the visual nature of the task usually leads someone to realize that if 1 ball and 1 jersey are $32, then 2 balls and 2 jerseys are $64. Then, students can determine that a jersey is $22 (86-64). I have had a room full of kids spontaneously burst into applause as students realize the efficiency of this strategy. Eventually we connect this visual model with the abstract, algebraic representation of elimination with multiplication for solving systems of equations.

Web 2.0 in the Mathematics Classroom

from November 2014 from Mathematics Teaching in the Middle School

A key characteristic of successful mathematics teachers is that they are able to provide varied activities that promote student learning and assessment. Web 2.0 applications can provide an assortment of tools to help produce creative activities. The term *Web 2.0* designates Internet applications that provide a context in which students create, collaborate, and communicate. The Internet technology is not unique, but the applications that are accessed and used are interactive and go beyond static browsing. A Web 2.0 tool enables the student to enter data and create multimedia products using text, graphics, audio, and video. The possibilities for creativity and variety are endless.

The Standards for Mathematical Practice (SMP) in the Common Core State Standards include a strong emphasis on student reasoning and sense making and on demonstrations of understanding (CCSSI 2010). In describing mathematical proficiency, these standards recommend that students should work collaboratively, explaining and discussing concepts to refine understanding (CCSSI 2010, SMP 2 and 3, p. 6). Students should model and apply their mathematics knowledge (SMP 4, p. 7) and use technological tools and communicate their understanding precisely (SMP 5 and 6, p. 7). Web 2.0 tools can be useful in structuring a variety of learning experiences to enable the development of students' habits of practice, as recommended by the Standards for Mathematical Practice. In the following activities, students interact with the mathematics in a creative and collaborative context and communicate their understanding both individually and in groups while working on projects.

THE GLOGSTER PROJECT

Glogster® is a Web 2.0 tool that is used to create a *glog*, which is "an interactive visual platform in which users create a 'poster' containing multimedia elements including text, audio, video, images, graphics, drawings, and data" (Glogster™EDU). The tool uses a simple drag-and-drop interface that is easily mastered by students of many ages and learning styles. It can be useful in not only a lower-level general math class but also an advanced course. The Glogster (http://edu.glogster.com) free version was used for this project. A commercial version includes additional features.

The Glogster Project was implemented in two algebra classes in a rural southern middle school. Each class included about twenty students who were in eighth and ninth grade. The students worked in groups of three or four to create a multimedia representation of an assigned algebra topic. This project was assigned late in the school year as a way to further examine major topics already studied in the course, including the Pythagorean theorem, the distance and midpoint formula, scatterplots, and the quadratic formula. The purpose of the activity was to engage students in revisiting and modeling their understanding of the concepts and to broaden and deepen their understanding. The group assignment was used to promote a discussion of the concepts.

The students were first oriented to Glogster by viewing examples of glog posters and learning how to use the software. Preparing the glog was a two-step process. Students first determined their content, and then created the design. The assignment stipulated that students create a glog for their topic containing the following content features:

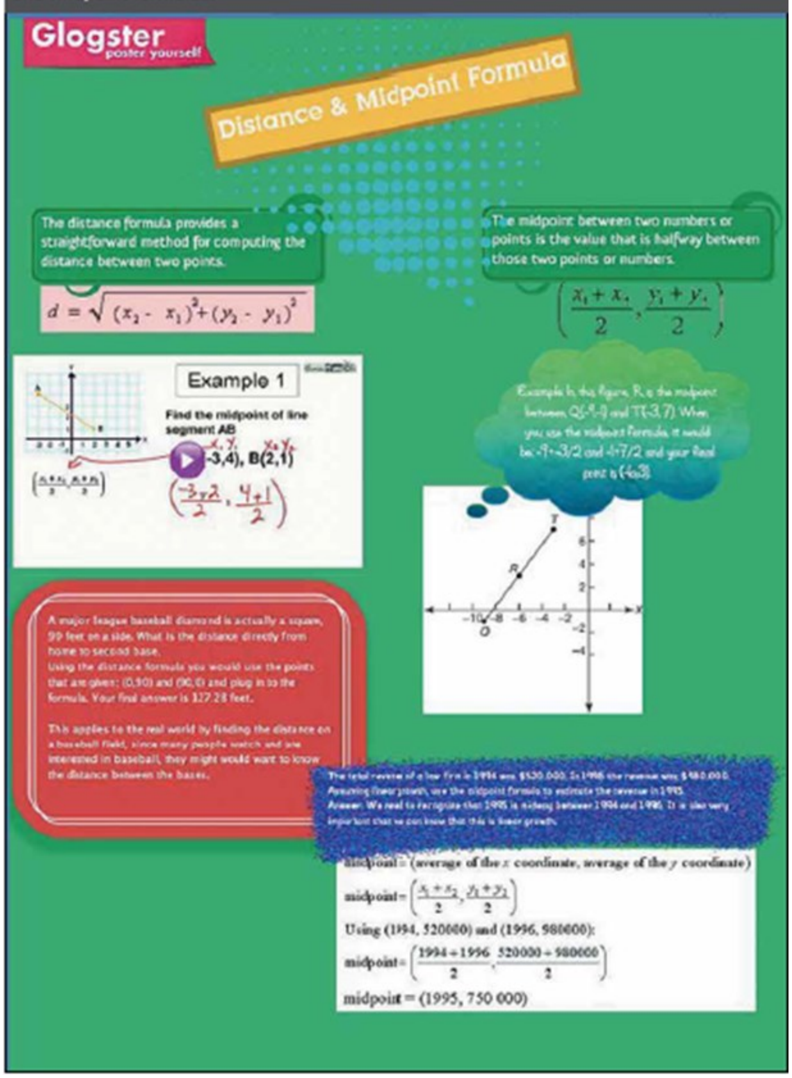
- An introduction of the topic and basic definitions
- Problems represented algebraically and graphically
- A statement to show where the topic fits in the "big picture" of mathematics
- An example of a real-world problem

Each group planned extensively. Students began by discussing the topic, definition, model, and application. Rich discussion was evident as students debated and constructed a group understanding of the topic. An informal storyboard was used to encourage students to take notes and keep track of their ideas.

Once the ideas were in place, the groups decided on the design features that they would incorporate into their glog. They were asked to include text (words, equations, algebraic expressions), images (pictures, graphs), and audio/video (an original podcast, a link to an online video, a link to their original video uploaded to YouTube). They could select from a wide variety of color schemes and could arrange elements as they chose. Computer lab time was allocated so that the groups could complete the design process. Video cameras and digital audio recorders were available for original media, which was encouraged. Each group completed a glog that was presented to the class at the end of the two-week project. Assessment was based on a rubric.

The glog in **figure 1** described the distance formula and the midpoint formula. It included graphs, formulas, video of a problem under construction (from SchoolTube®), and two applications: a baseball diamond and finance. Another glog, shown in **figure 2**, explored scatter plots and included several graphs, definitions, applications, and video calculator instructions.

Fig. 1 This glog demonstrated a group understanding and application of the distance and midpoint formulas.



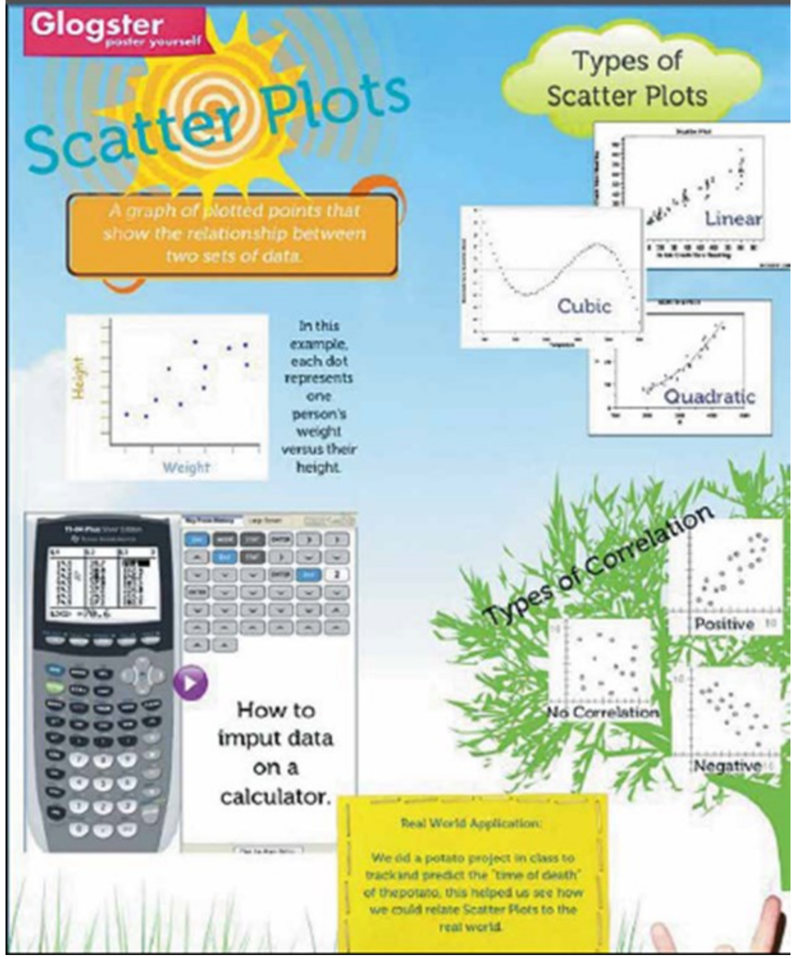
RESEARCH STUDY

The research study spotlighted here examined the effect on attitudes toward mathematics when the Glogster activity was used in a middle school algebra class. Participants in the study were thirty students in two classes. Data were not included for students who failed to return consent forms. The data collection involved both a presurvey and a postsurvey of student attitudes toward mathematics. A focus group discussion with students at the end of the study collected comments about the project activities.

Results from both the pre-attitude and post-attitude measure showed that the students were more positive toward mathematics at the end of the study. The focus group discussion indicated that students enjoyed the activities, liked the active involvement, and found it motivating. One student said, “It is better than taking notes because then I get tired and I zone out. With this, you actually want to focus and figure out what you want to do because you have to present it and you want it to be good. And it will reflect on you.” Comments indicated that the presentations were a source of motivation for many students.

Students also reported that they enjoyed the multimedia, especially the color and the videos: “I like it when we get to do different colors and we got to personalize it and that made it pop out.” They said that they appreciated the collaboration because they found it was “cool to see everyone’s view on the math and their applications.” The students further revealed that they were quite pleased with their creations. One student said, “This makes math real and fun. I understand it better now.” An evaluation of their projects confirmed that they were able to accurately model the mathematical concepts, including real-world problems, in varied representations.

Fig. 2 Scatter plots were used appropriately by the group that produced this glog.



One disappointment was that groups used online video material rather than creating their own videos. The Khan Academy® (https://www.khanacademy.org) site was appealing to students, as were other videos available in SchoolTube (http://www.schooltube.com). Although students knew how to create a video or a podcast, this create-your-own video was only an option, and students chose the finished products that were available online. This may have occurred because of their lack of experience, thus making them unwilling to take a chance on original video. In future projects, additional instruction will be given on creating video, and an original video or podcast will be required.

Conclusions from the research component involved attitudes, engagement, and collaboration. From evidence based on survey results and comments, students’ attitudes toward mathematics showed improvement after the project. Their comments confirmed that they were engaged and interested. They indicated that the group collaboration was important in helping them talk about the content and understand it better. They felt that they learned mathematics, and they enjoyed it, as well. The Glogster tool was effective in providing a context for students to create (model), collaborate (discuss), and communicate their understanding.

OTHER WEB 2.0 TOOLS

Other creative, interactive Web 2.0 tools offer similar benefits and are briefly described below.

Wordle

An application that generates "word clouds" or pictures composed of words is the principle behind Wordle™ (Feinberg; http://www.wordle.net). The relative size of the words is proportional to their frequency in the chosen text. In addition to being fun and colorful, a Wordle is an effective way to assess student understanding and emphasize vocabulary. It can be used for guided reflection and discussion to promote a deeper understanding of the vocabulary. For example, student pairs were asked to generate a ten-word vocabulary list once they had finished a unit on equations. These lists were entered into the Wordle application. **Figure 3** shows one resulting graphic. Students were asked to discuss the Wordle in their groups and decide whether they agreed with the vocabulary terms that were identified as most important by the class. This guided reflection resulted in deeper learning of the vocabulary. A similar activity would be appropriate to emphasize the vocabulary in any topic or at any level.

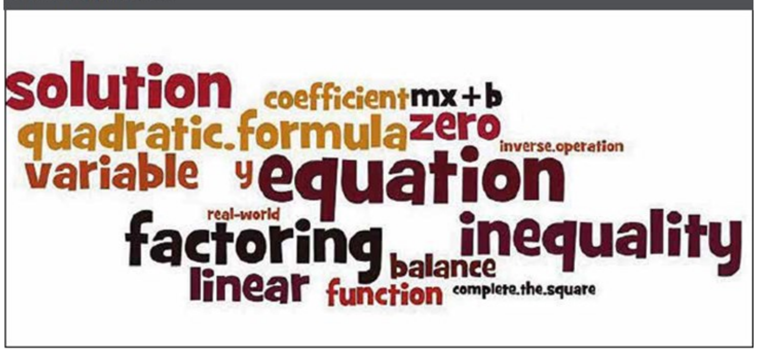


Fig. 3 The word *equations* provided the impetus for this student-created Wordle. The assignment asked students to generate ten associated words and plug them into the Wordle application.

Poll Everywhere

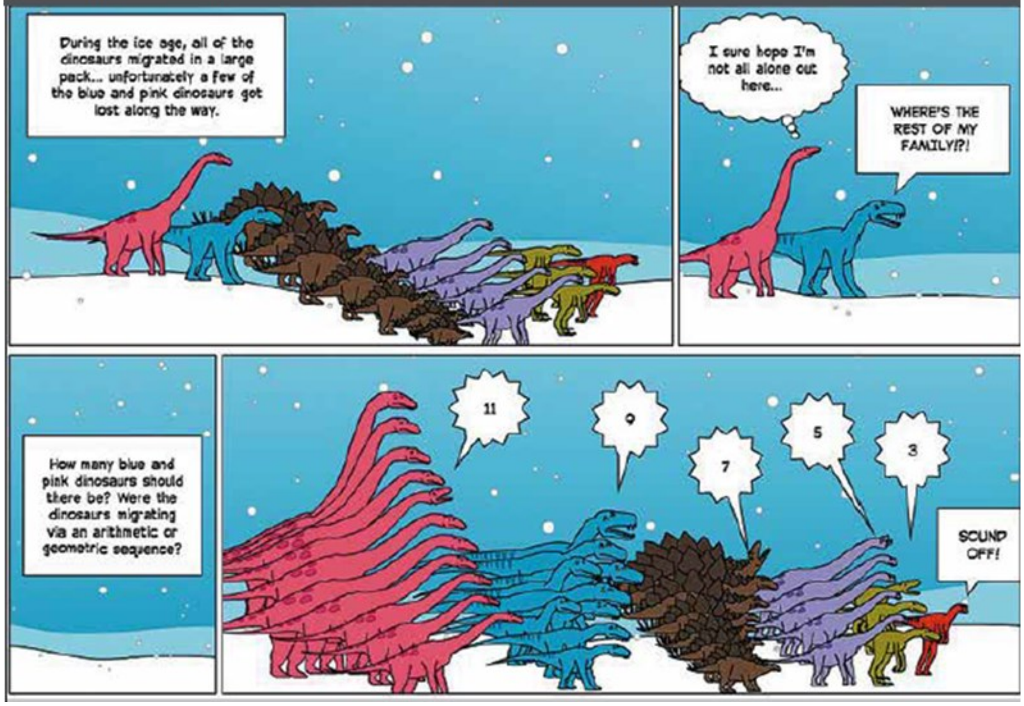
For a quick and easy way to collect data from students, use Poll Everywhere (http://www.polleverywhere.com). This application allows the teacher to set up a survey, and students can respond either by computer or by text message. (Yes, cell phones in the classroom!) The survey questions may be true or false, multiple choice, or open ended. Poll Everywhere will then display the results instantaneously, either as a table, a graph, or as text. The text can be transferred to a Wordle or simply used for discussion. This tool provides formative assessment that is similar to personal response systems, known as "clickers," which involve considerable expense and set up. The only requirement for Poll Everywhere is that each student or group has access to a smartphone or computer.

Pixton

To enable students to create their own comic strips using a simple click-and-drag interface, look up the online tool called Pixton® (http://pixton.com). This tool allows students to write their own application problems. **Figure 4** shows a simple Pixton example that students constructed in response to being asked to generate an arithmetic or geometric sequence of their own. Students then presented their comic strips to the class the next day, and the class discussed the problems. Problem posing has long been recognized as a motivating instructional practice in mathematics (e.g., Polya 1957), requiring students to reflect on the mathematics and model it in a real-world context. Pixton provides a graphic, electronic interface that broadens this traditional activity using a twenty-first-century learning tool.

Voki

Fig. 4 This Pixton, or comic strip, was constructed by a student group in response to an assignment to produce an arithmetic or geometric sequence of their own.



To create podcasts and customize characters that speak, students can access Voki® (http://www.voki.com/). They can choose a character, a background, and a voice or accent or use their own. Students might create a series of vokis to explain a concept or give examples or steps for solving a problem or understanding the context of a problem. In any of these uses, the student or group must reflect and plan how to articulate their understanding through a podcast. A sample Voki created by a student group shows a character stating the steps for solving a word problem. The group discussed the problem at length and planned carefully so that all members were in agreement and could justify each step. The final product can be accessed at http://www.voki.com/pickup.php?scid=7151891&height=267&width=200. This activity provided a different communication medium—audio—which involved both reflection and discussion as the group had to articulate about and agree on their understanding. The presentation could be given to the class or collected and evaluated by the teacher as formative or summative assessment data.

Trading Card

To create a product that resembles a baseball card, including graphics and text, students can access the Trading Card application (http://www.bighugelabs.com/deck.php). This online tool could be used, for example, to create a project to study the history of mathematics. Students can select a mathematician and create his or her own card. Another project involved student groups interviewing people in the community and making a card to show how mathematics was used in their work. **Figure 5** shows a student-created card for a police officer. In this project to facilitate connections between mathematics and the real world, the cards created by class groups were displayed on a bulletin board titled, "When are we ever going to use this?"

EduCreations

An online whiteboard called EduCreations® (http://www.educreations.com) allows the user to write and record audio. It produces Flash Videos similar to those found at the Khan Academy site and is an iPad® application that also works in any computer browser. This tool can be used by the teacher to produce online learning materials. However, it is most valuable when accessed by students. Student groups can be assigned a problem and can write on the electronic whiteboard as they explain what they are doing and why during their presentation via EduCreations. They can also import graphics, such as a grid for constructing a graph. The resulting video can be presented to the class or it can be viewed later by the teacher for assessment.

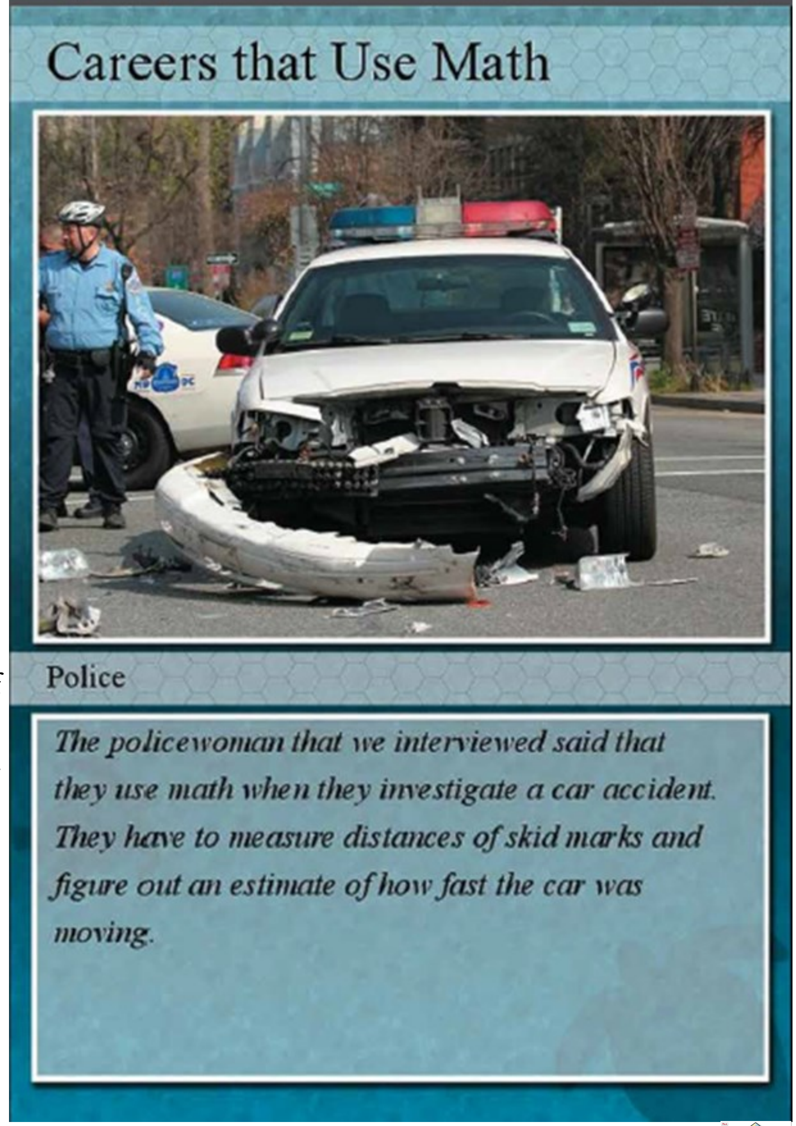


Fig. 5 The question, "When are we ever going to use this?" resulted in this Trading Card generated by a student to show connections between math and the real world.

TOOLS FOR STUDENTS

The activities described here show examples of Web 2.0 tools that can be used effectively in a variety of mathematics classes. They can assist students in creating, collaborating, and communicating understanding. It is important to remember that although a tool might be good for the teacher, it is almost always better to put it in the hands of the students. The maximum benefit will occur when students create and communicate a model of their understanding in a hands-on collaborative environment that also engages them mathematically.

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About the author: **Leah P. McCoy**, mccoy@wfu.edu, teaches mathematics education at Wake Forest University in Winston-Salem, North Carolina. She is interested in uses of technology in mathe-

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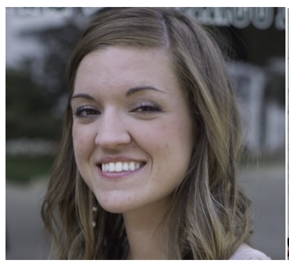

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